1.)

a) $\sigma = 50\ S \cdot m^{-1}$ $\qquad h = 300\,km$

$$n = 10^{6}\ cm^{-3} = 10^{12}\ m^{-3}$$

$$\tau = \frac{m\sigma}{ne^2} = 0.0018\ s$$

b)

$$\Delta E_y = \mu\varepsilon \ddot{E}_y + \mu\sigma \dot{E}_y = (-\mu_0\varepsilon_0\omega^2 + i\mu_0\sigma\omega)E_y = \lambda^2 E_y$$

$\mu = \mu' + i\mu'' = \mu_0\mu_r = \mu_0$ $\qquad \omega = 2\pi f = 2\pi \cdot 100 \cdot 10^6 = 6.28 \cdot 10^8\,s^{-1}$

$\varepsilon = \varepsilon_0$

$$\lambda^2 = -\mu_0\omega^2\left(\varepsilon_0 - i\frac{\sigma}{\omega}\right) = -\mu_0\varepsilon_0\omega^2\left(1 - i\frac{\sigma}{\varepsilon_0\omega}\right)$$

$$= -\frac{\omega^2}{c^2}\left(1 - i\frac{\sigma}{\varepsilon_0\omega}\right) \doteq -4.38 \cdot (1 - i\,8992) \doteq i\,39386$$

$$\Rightarrow \lambda = \sqrt{i\,39386} = 198 \cdot \sqrt{i} = \frac{198}{\sqrt{2}}(1+i)$$

c)

$$\lambda = i\frac{\omega}{c}n \qquad n = \frac{1}{i}\frac{\lambda c}{\omega} = \frac{1}{i}\cdot\lambda \cdot 0.478 = -i\lambda \cdot 0.478 =$$

$$= \frac{198}{\sqrt{2}} \cdot 0.478\,(-i+1) = 66.9\,(1-i)$$

d)

$$Z = \sqrt{\frac{\mu_0}{\varepsilon_0 - i\frac{\sigma}{\omega}}} = \sqrt{\frac{\mu_0}{\varepsilon_0}\left(\frac{1}{1 - i\frac{\sigma}{\omega\varepsilon_0}}\right)} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \sqrt{\frac{1}{1 - i\frac{\sigma}{\omega\varepsilon_0}}} =$$

$$\doteq 377\,\Omega \cdot \sqrt{\frac{1}{-i\,8992}} = 377\,\Omega \cdot \sqrt{i \cdot 0.000111} = 377 \cdot 0.01 \cdot \frac{1}{\sqrt{2}}(1+i)$$

$$= 2.81\,\Omega\,(1+i)$$

e)

$$n^2 = 1 - \frac{\sigma}{\varepsilon_0 \omega^2 \tau}$$

Cutt-Off $\quad n^2 = 0 \Rightarrow \varepsilon_0 \omega^2 \tau = \sigma$

$$\omega^2 = \frac{\sigma}{\tau \varepsilon_0} = \frac{ne^2 \tau / m}{\tau \varepsilon_0} = \frac{ne^2}{\varepsilon_0 m}$$

$$\omega = \omega_p = \sqrt{\frac{ne^2}{\varepsilon_0 m}} = 56.4 \cdot 10^6 \, s^{-1}$$

$$\Rightarrow f_{Cutt\text{-}off} = 8.97 \, MHz$$