

2004

Simulationsrechnungen zum Niederenergie-Ionenspeicher

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Inhalt:

- Magnetische toroidale Systeme
- Geometrie
- Magnetfeld
- Teilchensimulation
- Ausblick

Magnetische Systeme mit toroidaler Symmetrie

- Gyrationbewegung

$$\omega_g = qB / m$$

- Drift

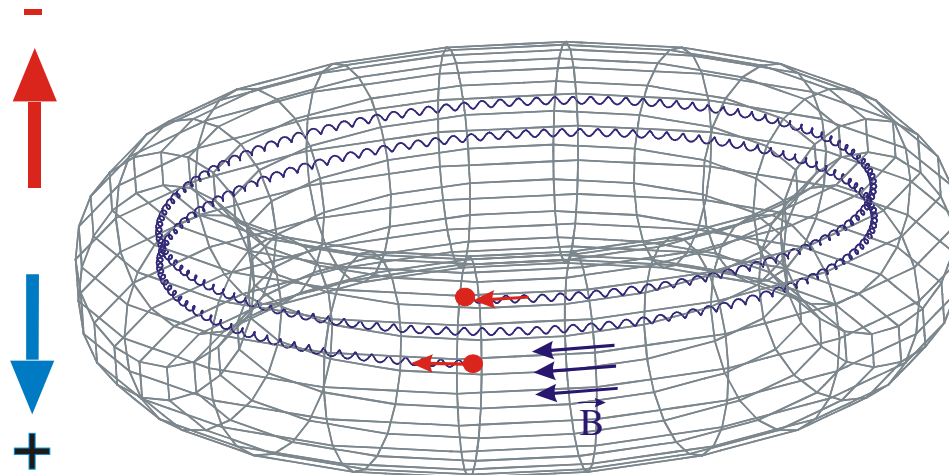
$$\vec{F} \times \vec{B}$$

$$\vec{F} = \vec{F}_c + \vec{F}_{\nabla B}$$

$$\vec{F}_c = mv_{\parallel}^2 / R \cdot \vec{e}_r$$

$$\vec{F}_{\nabla B} = -\mu \cdot \text{grad } \vec{B}$$

Ladungstrennung aufgrund der Drift



Theorie

Driftgeschwindigkeit und Gyrationfrequenz bei einer Feldkrümmung:

$$v_{d0} = \frac{m}{qBR} \left[v_{\parallel}^2 + \frac{v_{\perp}^2}{2} \right], \quad \omega_g = \frac{qB}{m}, \quad r_g = mv_{\perp} / qB$$

Bei konstanter Steigung $a = B_z / B_{\text{long}}$ in z-Richtung

$$v_d = v_{d0} \frac{1}{1 - a^2}, \quad \omega_g = \omega_{g0} \sqrt{1 - a^2}$$

Geometrie

Parameter:

$$\varphi = 72^\circ$$

$$h = 0.6 \text{ m}$$

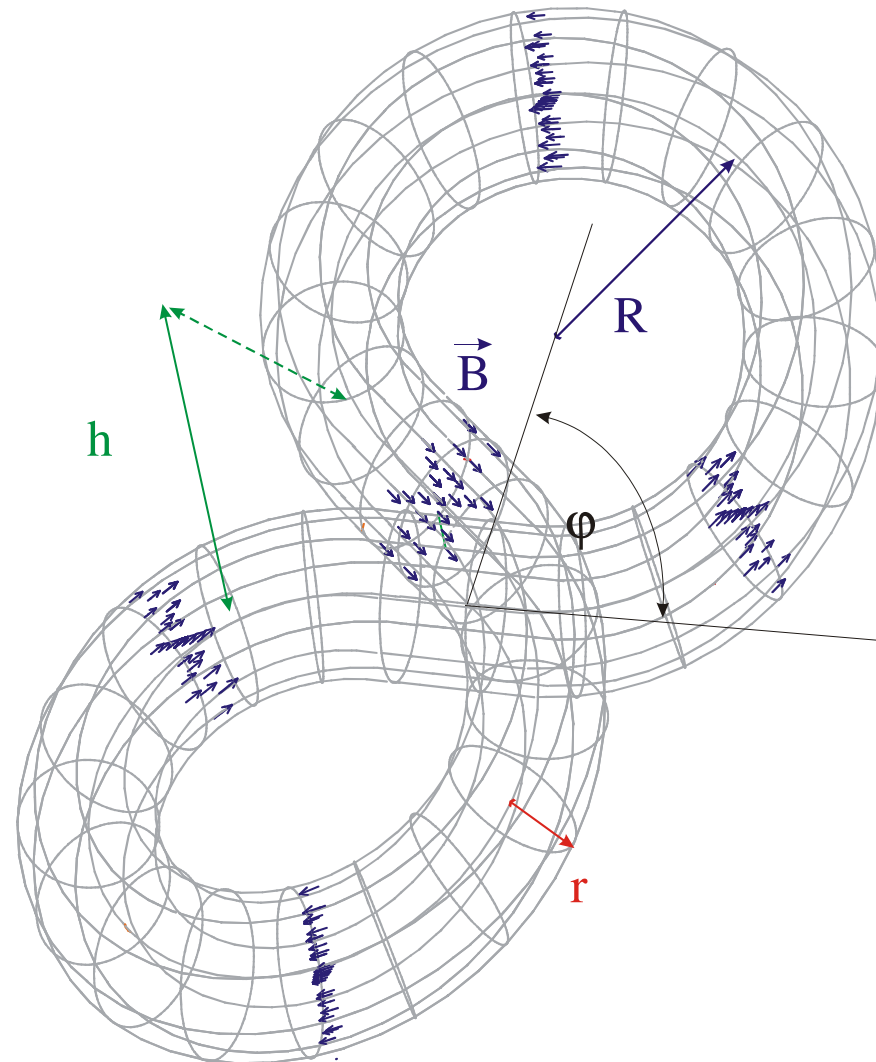
$$r = 0.25 \text{ m}$$

$$R = 1 \text{ m}$$

Steigung:

a) Linear

b) cos-Funktion



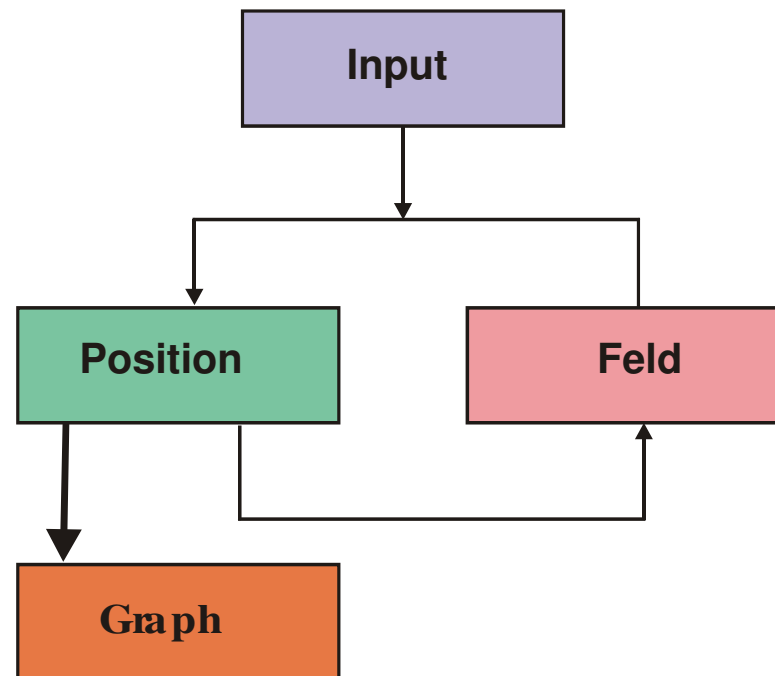
Teilchensimulation

- Feldberechnung – Biot-Savart Gesetz

$$\vec{B} = \int \frac{\mu I}{4\pi} \frac{d\vec{l} \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

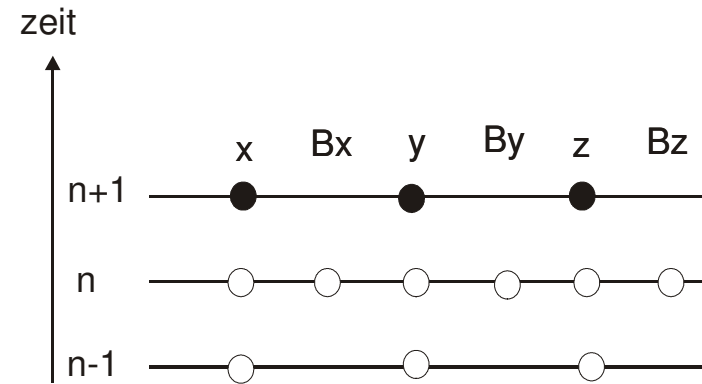
- Bewegungsgleichung

$$m \frac{d^2 \vec{r}}{d t^2} = q \vec{v} \times \vec{B}$$



Numerische Methoden

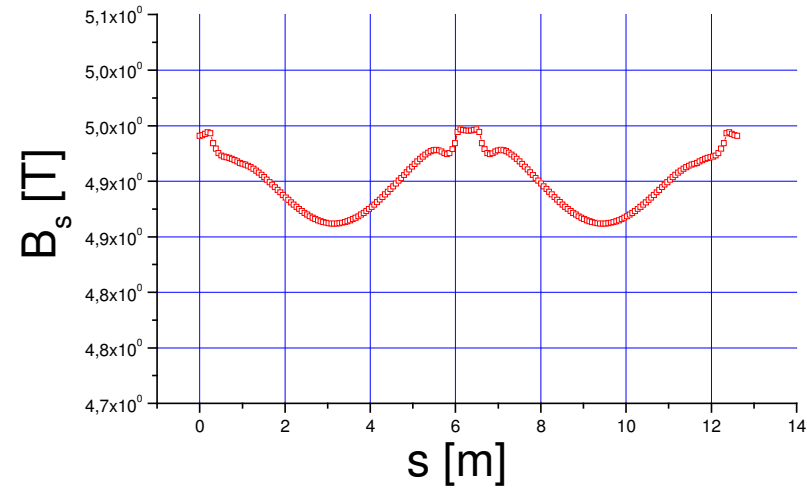
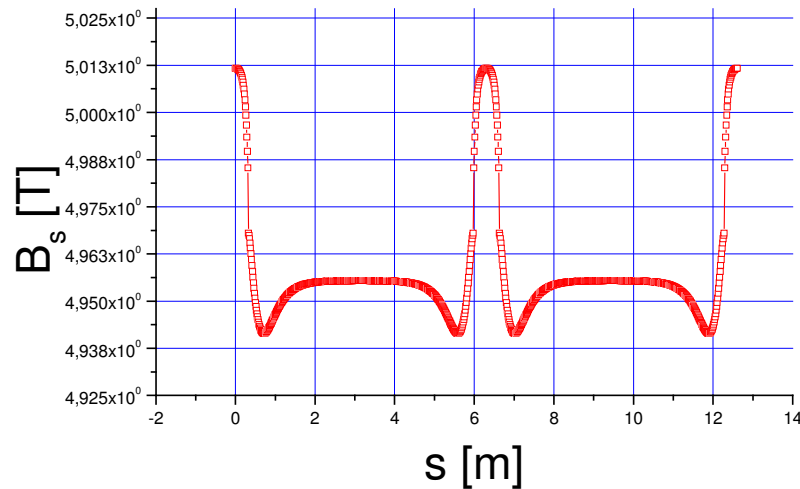
- Numerische Integration
- FDTD – Methode
„leap-frog“ Verfahren



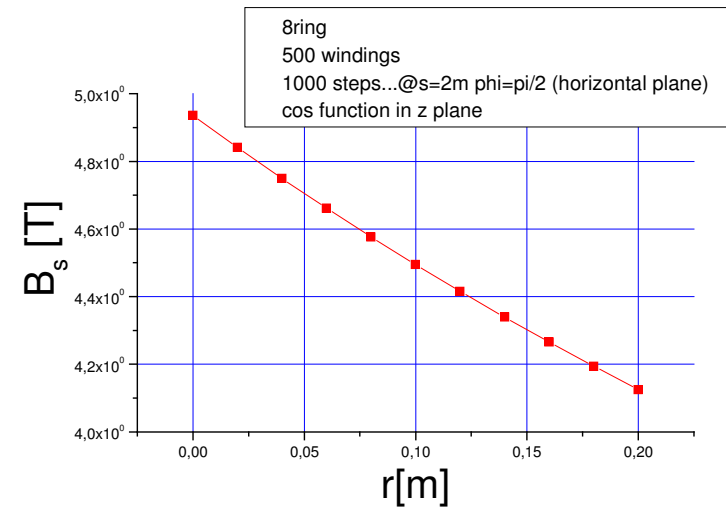
$$\frac{x_{n+1}}{\Delta t^2} - B_{z,n} \frac{q}{m} \frac{y_{n+1}}{2\Delta t} + B_{y,n} \frac{q}{m} \frac{z_{n+1}}{2\Delta t} =$$

$$\frac{2x_n}{\Delta t^2} - \frac{x_{n-1}}{\Delta t^2} - B_{z,n} \frac{q}{m} \frac{y_{n-1}}{2\Delta t} + B_{y,n} \frac{q}{m} \frac{z_{n-1}}{2\Delta t}$$

Magnetfeld

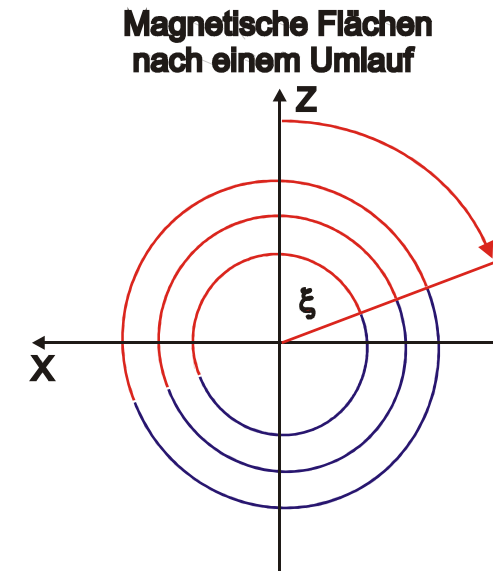
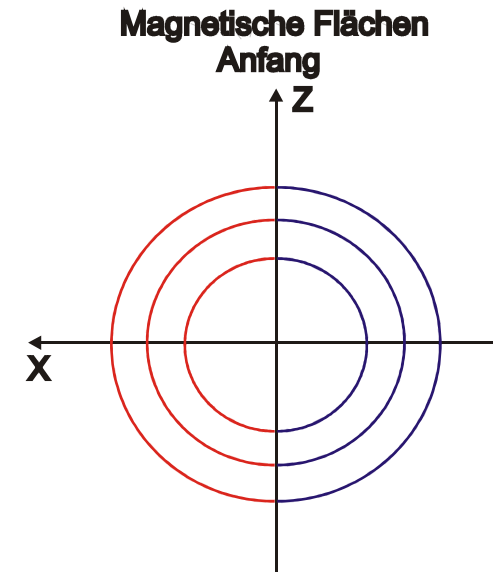
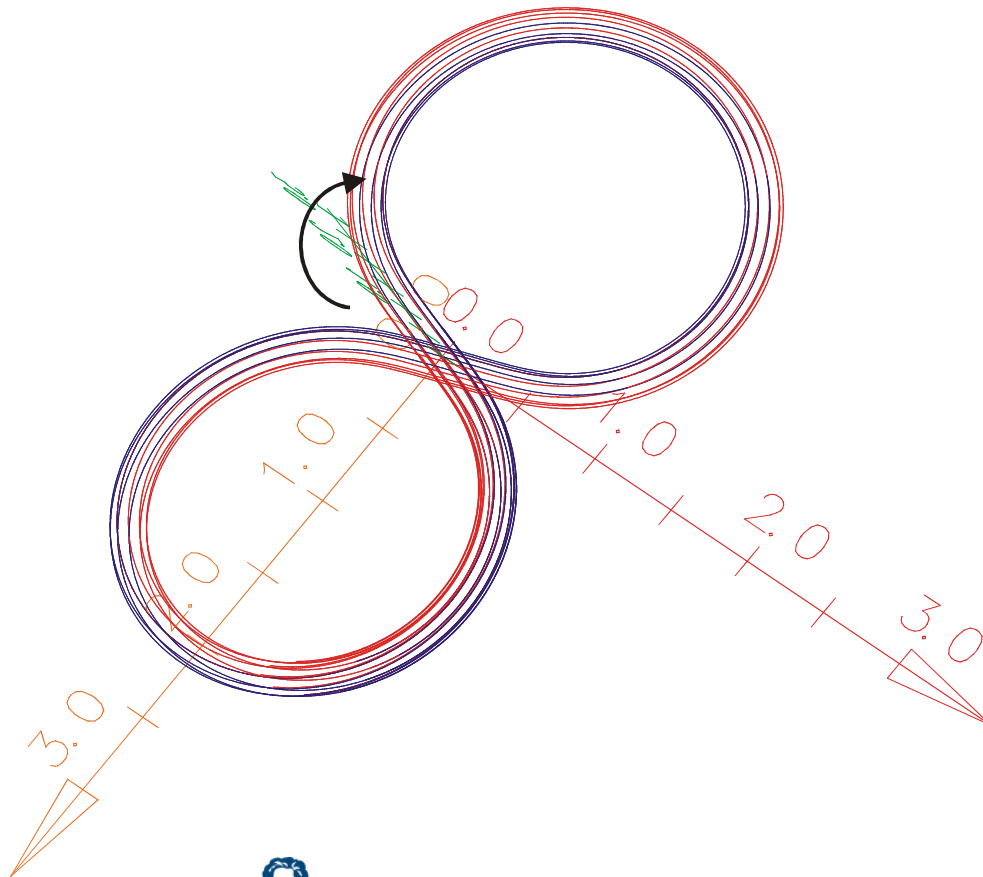


- Symmetrisch
- 2% Schwankung auf der Achse
- $1/R$ – Abhängigkeit in radialer Richtung

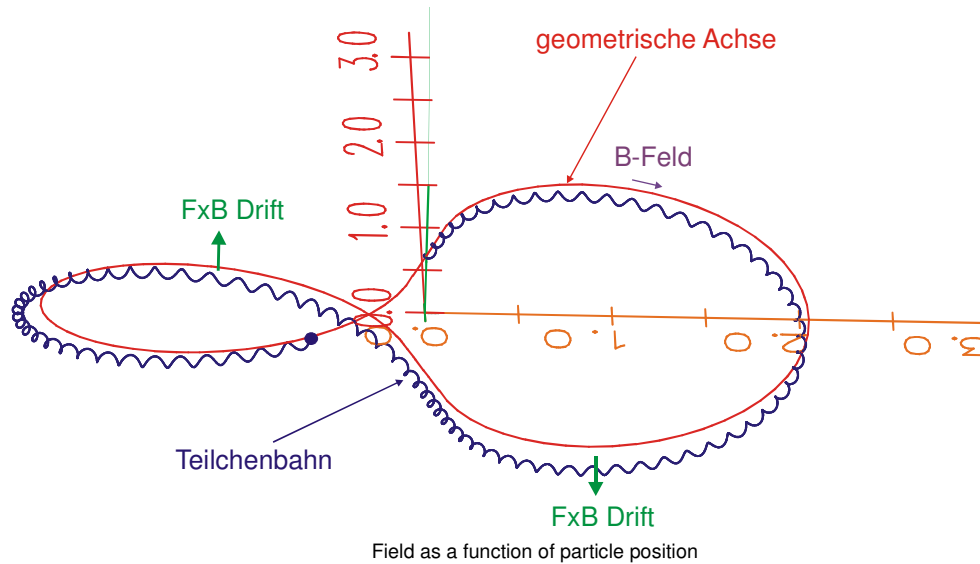


Rotation-Symmetrie

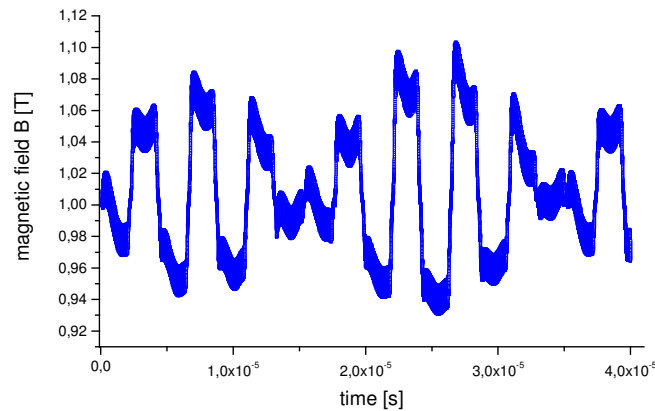
Drehung der magnetischen Flächen



Einzelteilchenbewegung



- Kurzzeit-Simulation (1-10 Umläufe)
- Stabile Bewegung
- Übereinstimmung mit der Theorie



Einschussebene

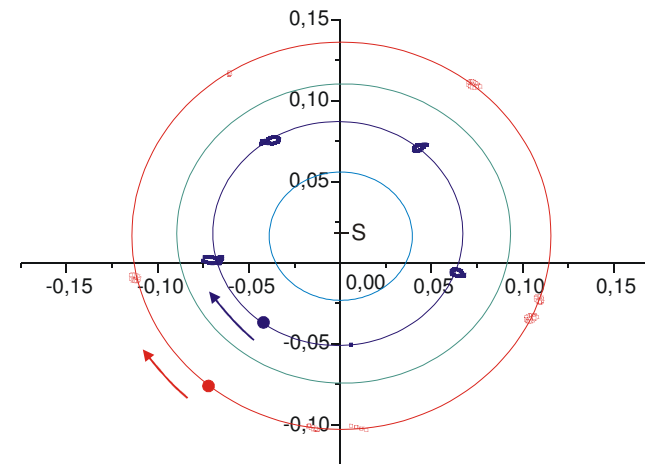
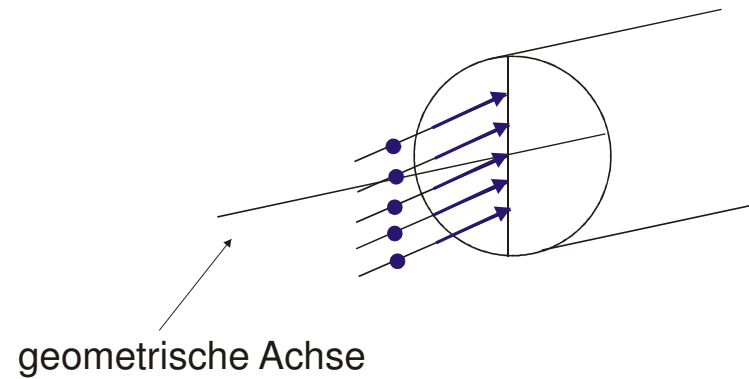
$$v_{\text{long}}/v_{\text{trans}} = \text{const}$$

$$W = 150 \text{ keV}$$

$$B = 5 \text{ T}$$

$$R = 1 \text{ m}$$

$$R_1 = 0.25 \text{ m}$$



Einschusswinkel

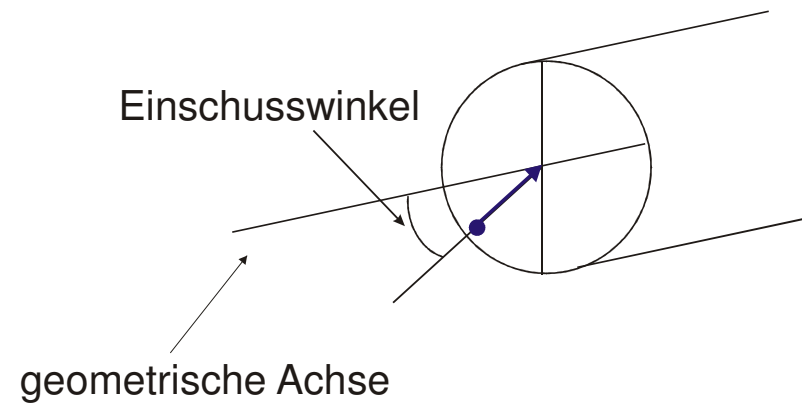
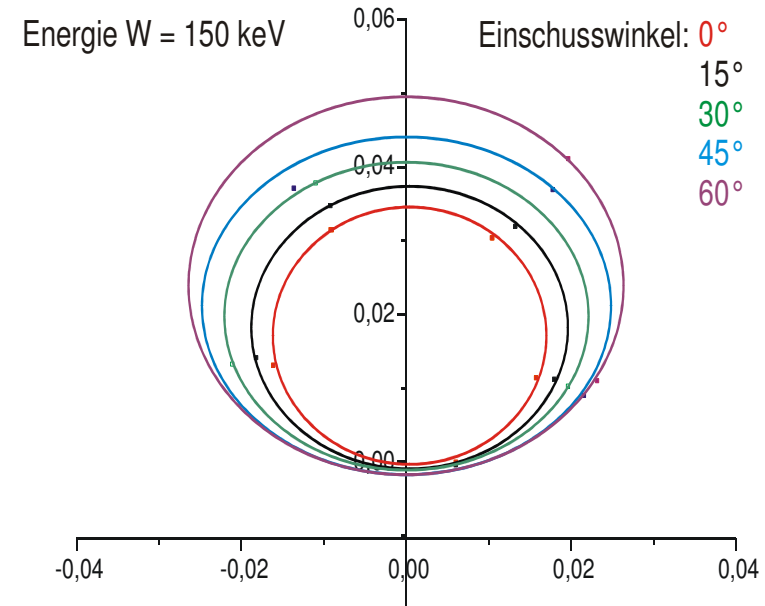
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Ausblick

- Mehrteilchensimulation
- Raumladung
- Form der Magnetspulen
- Stabilität

New parameter set

- $B=1\text{T}$, $r=0.25\text{m}$, $R=0.5\text{m}$

- Electrons

$$\omega_g = qB / m = 1.76 \cdot 10^{11} \text{ Hz}$$

$$r_g = m / qB \cdot v_{\perp} = 5.7 \cdot 10^{-12} \cdot v_{\perp} [\text{m}]$$

$$v_{d0} = \frac{m}{qBR} \left[v_{\parallel}^2 + \frac{v_{\perp}^2}{2} \right] = \frac{5.7 \cdot 10^{-12}}{\sqrt{1 - \frac{v^2}{c^2}}} \left[v_{\parallel}^2 + \frac{v_{\perp}^2}{2} \right]$$

$$v_E = \frac{1}{B^2} [E \times B]$$

- Protons

$$\omega_g = qB / m = 9.6 \cdot 10^7 \text{ Hz}$$

$$r_g = m / qB \cdot v_{\perp} = 1.04 \cdot 10^{-8} \cdot v_{\perp} [\text{m}]$$

$$v_{d0} = \frac{m}{qBR} \left[v_{\parallel}^2 + \frac{v_{\perp}^2}{2} \right] = 1.04 \cdot 10^{-8} \left[v_{\parallel}^2 + \frac{v_{\perp}^2}{2} \right]$$

Experiments and Theory of non-neutral plasma

- W. Clark (Maxwell Lab., California, USA, 1975)
 - relativistic electrons, densities $\sim 10^{10} \text{ cm}^{-3}$, potential well $\sim 300 \text{ kV}$
 - toroidal magnetic field, 7.8 kG, rise time 2 - 3.5 ms, + vertical field for stabilizing
 - $R=0.5 \text{ m}$, $r=0.081 \text{ m}$, Al shell 2.1 cm, vacuum 10^{-9} Torr
 - Diocotron modes $f \approx Q / 8\pi^3 Rr\epsilon_0 B$
 - 2 probes
 1. Circular stain-less disk mounted flush with the vacuum wall, but electrically insulated from it...terminated 50 Ohm cable to osciloscop...imaging charges detection
 2. High impedance voltage probe....0.5 mm wire covered with an insulated glass sheath except for the tip which is exposed to the electron cloud, measuring potentials up to $\sim 100 \text{ kV}$
 - oscillation after $200 \mu\text{s}$ \rightarrow ion resonance instability ?

- A. Mohri (Nagoya University, Japan, 1975)
 - 2 stage Marx generator (100 kV, 5 kJ)
 - Magnetic field ~ 7 kG, + vertical magnetic field
 - Lifetime of the beam current fluctuated from run to run...longest ~ 20 μ s, beam was not hollow
 - Vacuum 6×10^{-7} Torr, $I_{\text{current}}=300$ A, density ~ 8×10^8 cm $^{-3}$, potential well ~ 1.1×10^4 V
 - Radial electric field ~ 4×10^3 V/cm
 - Dump not observed in high gas pressure ~ 0.2 Torr
 - Current at the onset of the dump was nearly constant over a wide range of pressure
 - Ion resonance instability ? (Buneman, Levy, Daugherty)
 - Low ion density \rightarrow azimuthal wave mode $l=1$

$$\omega_e = n_e e / 2 \epsilon_0 B \cong \left(\Omega_E^2 + \frac{1}{4} \Omega_c^2 \right)^{\frac{1}{2}} - \frac{1}{2} \Omega_c = \Omega_i, \quad \Omega_E^2 = Z n_e e^2 / 2 \epsilon_0 m_i$$

- higher vertical field \rightarrow displacement to major axis \rightarrow excentricity \rightarrow diocotron oscillations \rightarrow hard-x-ray bursts \rightarrow second instability

- *Puravi Zaveri (Bhat, India, 1991)*
 - *Electrons should be lost due to curvature drift*
 - *But strong ExB drift ->rotation overcomes curvature drift*
 - *Electrostatic hoop forces tend to expand ring*
 - *Equilibrium -> closer to the inboard conducting shell*
 - *Strong toroidicity->strong distortion of the surfaces ->large ellipticity and triangularity*
 - *$B <40, 150>G$*
 - *Pressure 4×10^{-7} Torr, $I=150$ mA, density= 10^8cm^{-3} ,confinement time 2-2.5 ms*
 - *Theory->Only $3\mu\text{s}$ to reach the chamber wall due to (grad B) drift*
 - *Diocotron instability*
 - *Charge injection $15\mu\text{s}$, grad B drift $\sim 10^6$ m/s, ExB drift $\sim 10^8$ m/s*
 - *Single particle to collective behaviour ($I_{\text{critical}}=1$ mA)*

- *T.S.Pedersen (Colombia University, New York, USA,2002)*

-*Diocotron modes can be stabilized either by Landau damping*

$$R / (i\lambda_D) \cdot \sqrt{n / n_B} \ll 1, \quad \lambda_D = \sqrt{\epsilon_0 T_e / (e^2 n_e)}, \quad n_B = \epsilon_0 B^2 / (2m_e)$$

-*or by magnetic shear*

-*electrical current -> change in the magnetic field* $\delta B / B \approx (n_e / n_B)^2 \left(\frac{a}{c / \omega_c} \right)^2$

- *K. Avinash (Bhat, India,1991)*

- *drift surfaces shifted toward the major axis $\sim r/R(\omega_p/\omega_c)^2$*

- *in the absence of the conducting shell, the force along major radius could be balanced by an externally applied electric field*

- *question: as the walls of vessels are never perfect conducting, what happens to the shifted equilibrium in the presence of finite resistivity of the walls? \rightarrow resistivity destroy imaging charges \rightarrow grow of diocotron modes \rightarrow instability*

- M. R. Stoneking (Lawrence University, Wisconsin, USA, 2001)
- electron plasma, partial torus, horizontal electric field $E=5-10$ V/cm (larger than by Bhattacharyya ~ 1.4 V/cm for the same charge ~ 8.1 nC $\sim 5 \times 10^{10}$ electrons)
- Vacuum chamber 3mm Al, square poloidal cross-section, $R=0.5$ m
- Electron poloidal $E \times B$ drift frequency 240 kHz
- $B = 196$ G, typically energy $W=100-200$ eV
- Coils 24 evenly spaced bundles of 4 turns each, connected in series
- 2 interleaved sets (12 bundles each) are wound with opposing helicity \rightarrow net toroidal current = 0
- Electron source – 0.5mm tungsten spiral – 22 turns - $I=10.2$ A - $T=1900$ K
- Pressure 10^{-6} Torr, $I=150$ mA, density= 3.1×10^6 cm $^{-3}$, confinement time 0.1 ms
- At small electric field \rightarrow no evidence of trapping
- Some evidence of low frequency oscillation \rightarrow ion resonance \rightarrow possible diocotron modes \rightarrow modification with horizontal electric field

Magnetic coordinates for stellarator fields

- X.Bonnin (Greifswald, Germany, 2003)
 - Boozer-like coordinates \rightarrow field line trace $\rightarrow \chi_0 = \int \vec{B} d\vec{l}$
 - Definition of flux surfaces
 - Rotational transformation ι
 - Toroidal and poloidal currents flow

- F.Alladio (Frascati, Italy, 1995)
 - $d\chi_0 = \vec{B} d\vec{l}$
 - covariant representation $\vec{B} = \vec{\nabla} \chi_0 + \beta \vec{\nabla} \psi_T$ (assumption of scalar pressure MHD equilibrium $\rightarrow \vec{j}$ flows along flux surfaces $\rightarrow \vec{j} \cdot \vec{\nabla} \psi_T = 0$)
 - Magnetic field on flux surface $\vec{B} = \vec{\nabla}_{\parallel} \chi_0 \rightarrow$
 - Contravariant representation $\vec{B} = \vec{\nabla} \psi_T \times \vec{\nabla} \theta_0$