Simulation of non-neutral plasmas

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Riezlern 2007

Overview

- Debye length
- Figure-8 storage ring
- Status of developed codes
- Injection Experiment

Neutral Plasma

Number of particles in Debye sphere

$n\lambda_D^3 >> 1$

Debye length smaller than size of plasma

$\lambda_D < L$

Observed time scale longer than

$T > 2\pi/\omega_p$

Neutrality -> +=- (quasineutrality)

Non-Neutral Plasma

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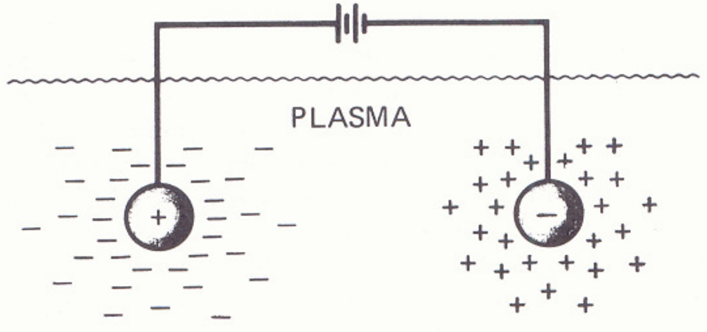
Debye length – neutral plasma

$$n_i = n_e = n$$

$$-\Delta\phi = -\frac{e}{\varepsilon_0}\delta(\vec{r}-\vec{r}^{\prime}) + \frac{ne}{\varepsilon_0}(1-\exp(\frac{e\phi}{kT}))$$



Debye shielding.

$$\frac{e\phi}{kT} << 1 \quad \Rightarrow \exp(\frac{e\phi}{kT}) \approx 1 + \frac{e\phi}{kT}$$

$$-\Delta\phi = -\frac{e}{\varepsilon_0}\delta(\vec{r}-\vec{r}^{\prime}) - \frac{ne}{\varepsilon_0}\frac{e\phi}{kT}$$

$$-\Delta\phi + \frac{1}{\lambda_D^2}\phi = -\frac{e}{\varepsilon_0}\delta(\vec{r}-\vec{r}^{\prime}) \quad \Rightarrow \quad \phi = \frac{-e}{4\pi\varepsilon_0(|\vec{r}-\vec{r}'|)}\exp(-\frac{|\vec{r}-\vec{r}'|}{\lambda_D})$$

Debye length – non-neutral plasma

$$-\Delta\phi = -\frac{ne}{\varepsilon_0}\exp\left(\frac{e\phi}{kT}\right)$$

$$-\Delta(\phi+\delta\phi) = -\frac{e}{\varepsilon_0}\delta(\vec{r}-\vec{r}\,') - \frac{ne}{\varepsilon_0}\exp\left(\frac{e(\phi+\delta\phi)}{kT}\right)$$

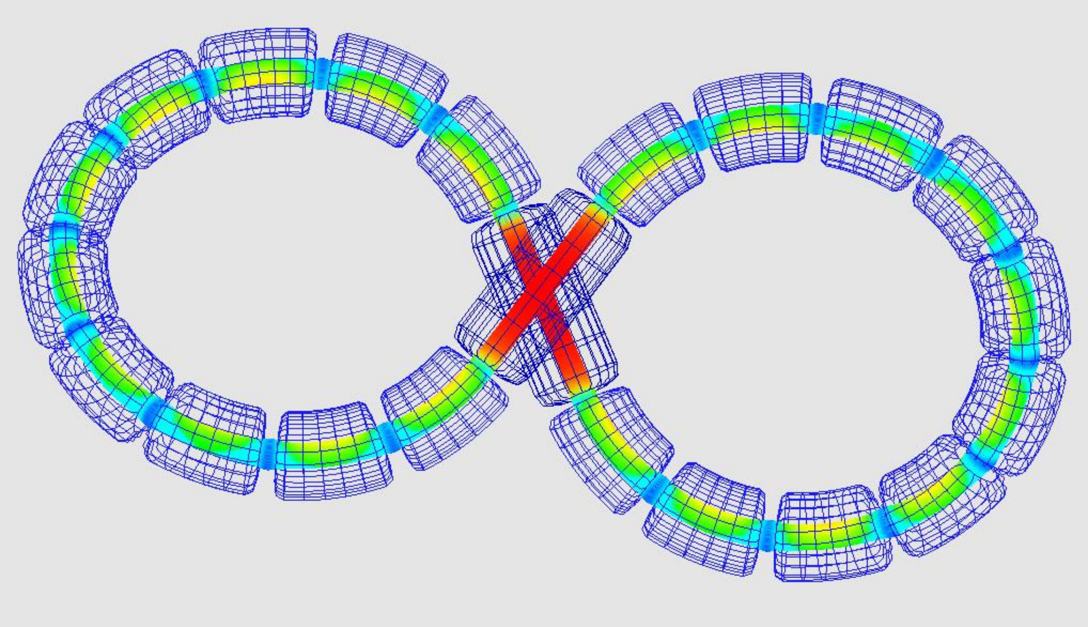
Poisson-Boltzmann equation
Electron plasma

$$\frac{e\delta\phi}{kT} << 1 \quad \Rightarrow \exp\left(\frac{e\delta\phi}{kT}\right) \approx 1 + \frac{e\delta\phi}{kT}$$

$$-\Delta\delta\phi = \frac{e}{\varepsilon_0}\delta(\vec{r}-\vec{r}\,') - \frac{ne}{\varepsilon_0}\frac{e\delta\phi}{kT}$$

$$-\Delta\delta\phi + \frac{1}{\lambda_D^2}\delta\phi = \frac{-e}{\varepsilon_0}\delta(\vec{r}-\vec{r}\,') \quad \Rightarrow \quad \delta\phi = \frac{-e}{4\pi\varepsilon_0(|\vec{r}-\vec{r}\,'|)}\exp\left(-\frac{|\vec{r}-\vec{r}\,'|}{\lambda_D}\right)$$

Figure-8 Storage Ring

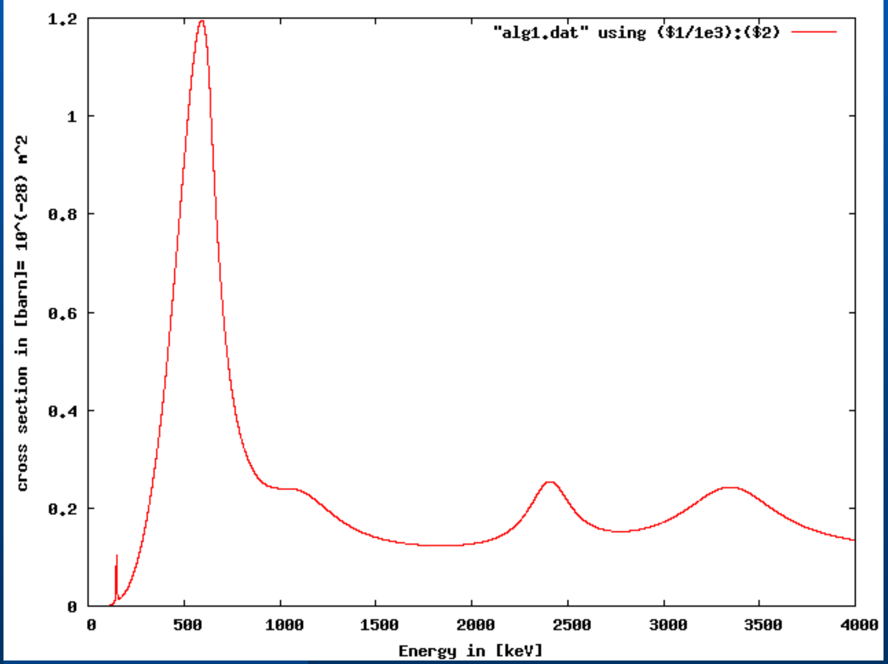


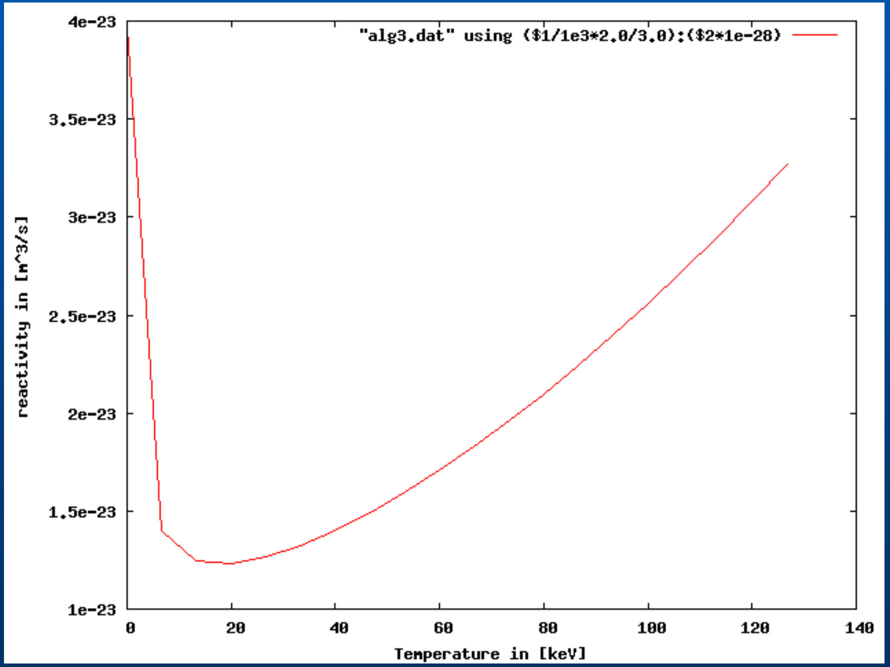
- R ~ 1m
- r ~ 0.2m
- L~10m
- 22 toroidal segments
- h ~ 1m
- B ~ 5T
- I ~ 10A

Proton-Boron

fusions reactions

$^{11}B+p \rightarrow 3\alpha$ (8.7MeV) fusion cross section $\sigma \sim 10^{-28} m^2$





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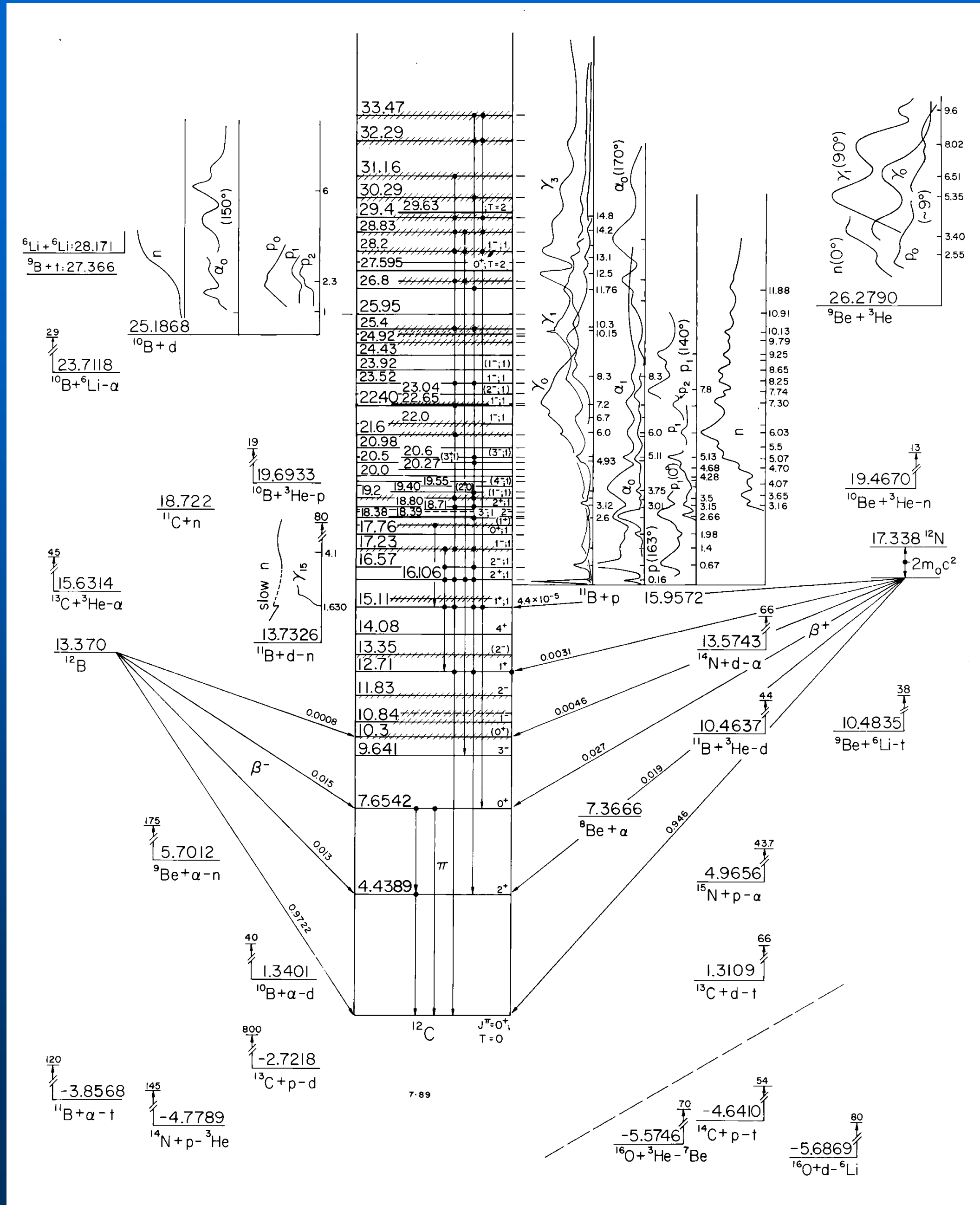
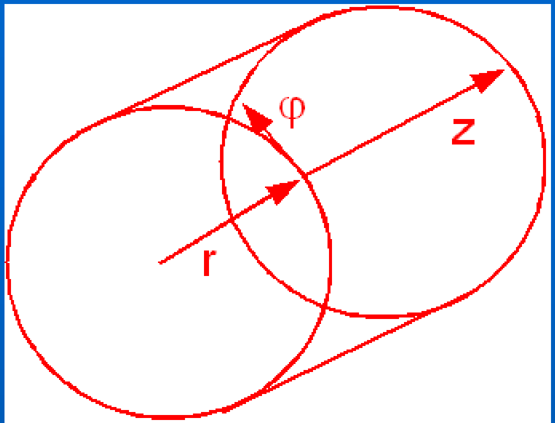


^{12}C
$J^{\pi}=0^+; T=0$
7.6542
4.4389
9.641
10.3
10.84
11.83
12.71
13.35
14.08
15.11
16.106
16.57
17.23
17.76
25.95
26.8
27.595
28.2
28.83
29.4
29.63
30.29
31.16
32.29
33.47
13.370
^{12}B
β^-
15.9572
$^{11}B+p$
17.338 ^{12}N
$2m_oc^2$
β^+
13.5743
$^{14}N+d-\alpha$
10.4637
$^{11}B+^3He-d$
7.3666
$^8Be+\alpha$
25.1868
$^{10}B+d$
26.2790
$^9Be+^3He$
19.4670
$^{10}Be+^3He-n$
10.4835
$^9Be+^6Li-t$
4.9656
$^{15}N+p-\alpha$
1.3109
$^{13}C+d-t$
-4.6410
$^{14}C+p-t$
-5.5746
$^{16}O+^3He-^7Be$
-5.6869
$^{16}O+d-^6Li$
$^6Li+^6Li$: 28.171
^9B+t: 27.366
23.7118
$^{10}B+^6Li-\alpha$
19.6933
$^{10}B+^3He-p$
18.722
$^{11}C+n$
13.7326
$^{11}B+d-n$
15.6314
$^{13}C+^3He-\alpha$
5.7012
$^9Be+\alpha-n$
1.3401
$^{10}B+\alpha-d$
-2.7218
$^{13}C+p-d$
-3.8568
$^{11}B+\alpha-t$
-4.7789
$^{14}N+p-^3He$
7-89

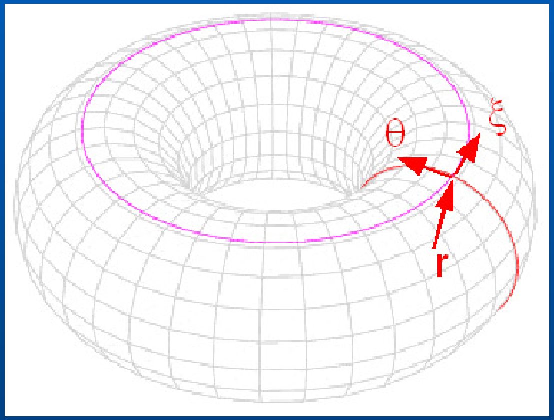
Cylindrical coordinates

r, φ, z



Toroidal coordinates

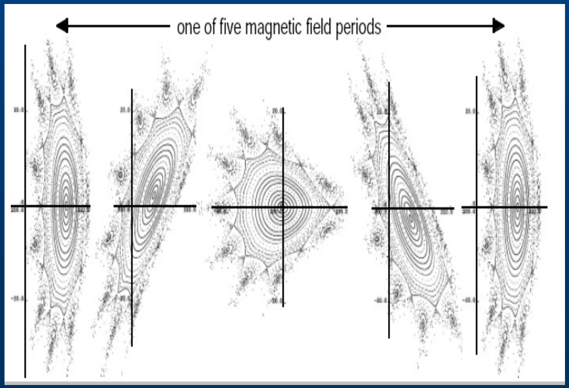
r, θ, ξ



Magnetic coordinates

(Clebsch, Boozer.....)

ψ, θ, ξ ψ, α, χ



Codes

- Field mapping – Biot-Savart solver

  (Predictor-Corrector method, Field-line integration – 1D information)
- Frequency decomposition – FFT (for every surface – 1D => 2D)
- Reverse mesh design – in new coordinates

  $\psi<0,1>$, $\theta<0,2\pi>$, $\xi<0,2\pi>$
- Poisson equation
- Guiding center drift motion

Poisson equation

Orthogonal basis

$$h_i = \sqrt{\left(\frac{\partial x}{\partial q_i}\right)^2 + \left(\frac{\partial y}{\partial q_i}\right)^2 + \left(\frac{\partial z}{\partial q_i}\right)^2}$$

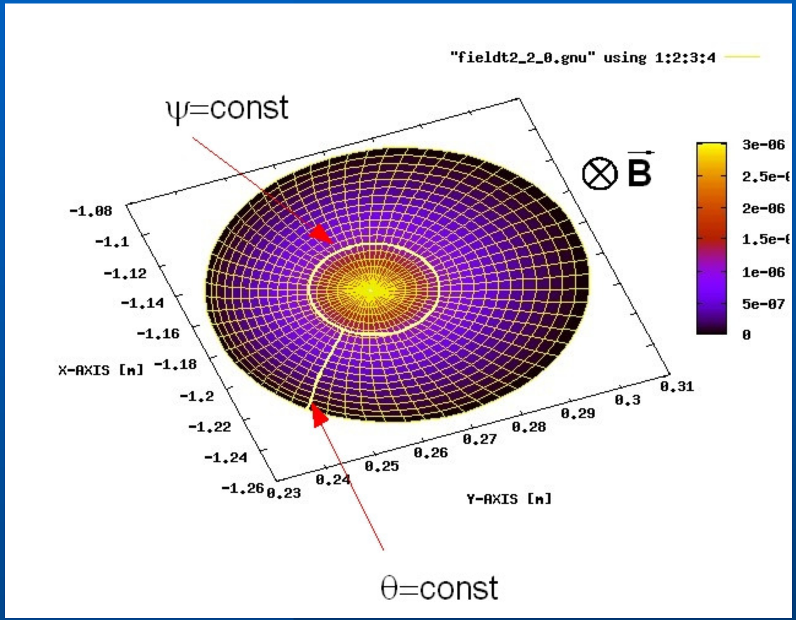
$$q = (\psi, \theta, \xi)$$

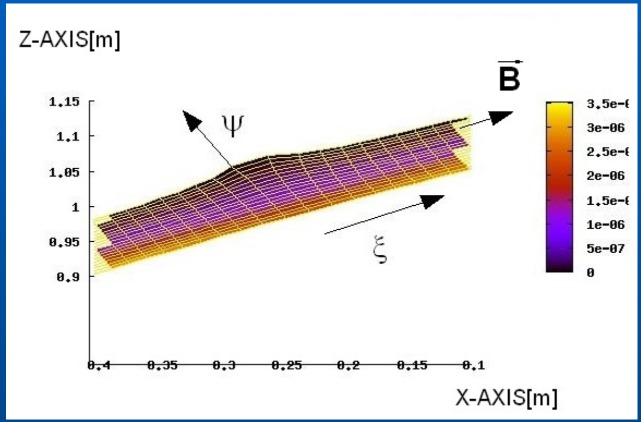
$$\Delta\phi = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial\psi}\left(\frac{h_2 h_3}{h_1}\frac{\partial\phi}{\partial\psi}\right) + \frac{\partial}{\partial\theta}\left(\frac{h_1 h_3}{h_2}\frac{\partial\phi}{\partial\theta}\right) + \frac{\partial}{\partial\xi}\left(\frac{h_2 h_1}{h_3}\frac{\partial\phi}{\partial\xi}\right)\right]$$

On axis – integral Gauss law

Numerical – iteration method, parallel implementation

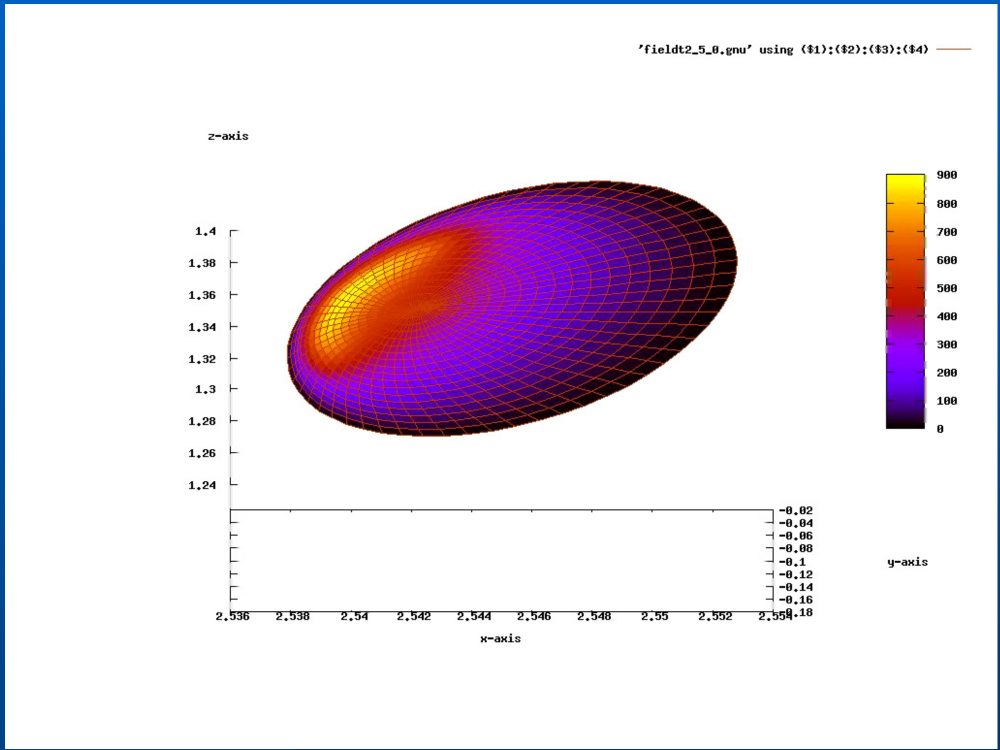
Electric Potential

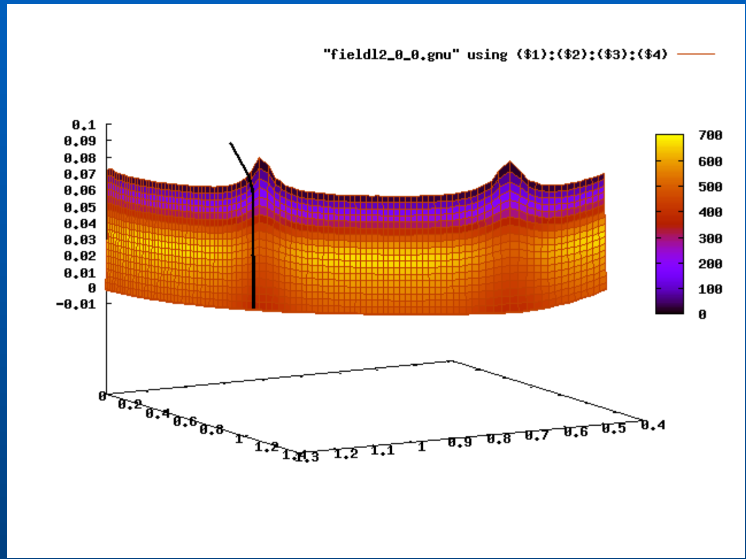




- Test charge on the magnetic axis

Potential Distribution





- Test charge off the magnetic axis

Drift motion equations

$$\frac{d\alpha}{dt} = -\frac{\partial\phi}{\partial\psi} - \left(\frac{1}{e}\mu + \frac{eB}{m}\rho_{\parallel}^2\right)\frac{\partial B}{\partial\psi}$$

$$\rho_{\parallel} = \frac{mv_{\parallel}}{eB}$$

$$\frac{d\psi}{dt} = \frac{\partial\phi}{\partial\alpha} + \left(\frac{1}{e}\mu + \frac{eB}{m}\rho_{\parallel}^2\right)\frac{\partial B}{\partial\alpha}$$

$$\chi = \int Bdl$$

$$\frac{d\chi}{dt} = \left(\frac{eB^2}{m}\rho_{\parallel}\right)$$

$(\alpha,\psi,\chi) \to (\theta,\psi,\xi)$

$\alpha = \theta - \iota\xi$ $\chi = g\xi$

$$\frac{d\rho_{\parallel}}{dt} = -\frac{\partial\phi}{\partial\chi} - \left(\frac{1}{e}\mu + \frac{eB}{m}\rho_{\parallel}^2\right)\frac{\partial B}{\partial\chi}$$

Drift Hamiltonian

- **In vacuum field – without magnetic self fields**

$$H = \frac{1}{2}\rho_{\parallel}^2 \frac{eB^2}{m} + \mu \frac{B}{e} + \phi$$

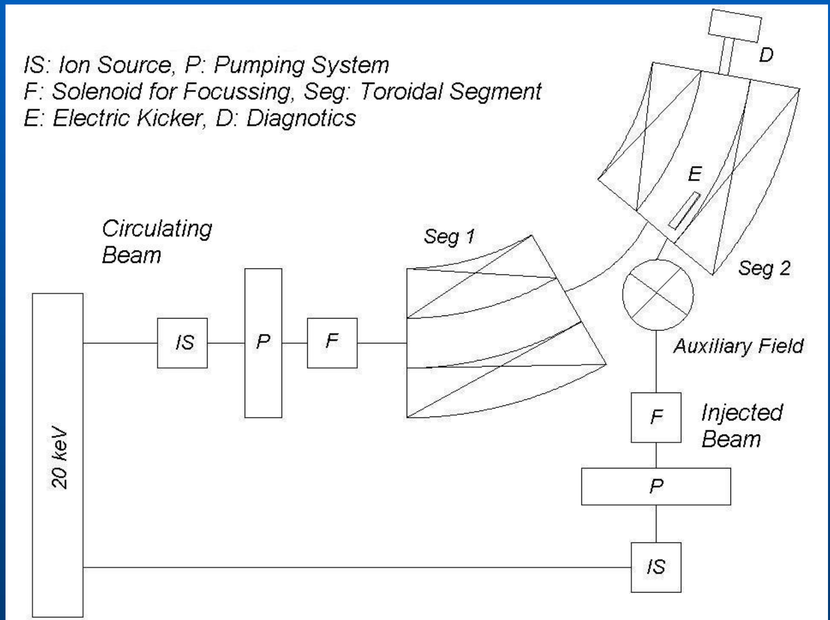
$$\frac{d\chi}{dt} = \frac{\partial H}{\partial \rho_{\parallel}}, \quad \frac{d\rho_{\parallel}}{dt} = -\frac{\partial H}{\partial \chi}$$

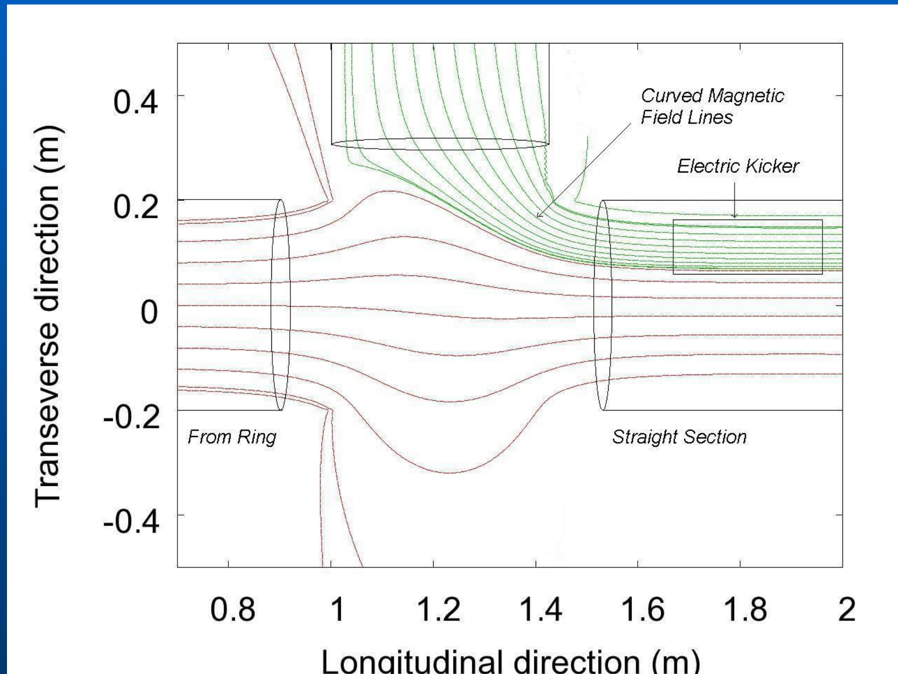
$$\frac{d\psi}{dt} = \frac{\partial H}{\partial \alpha}, \quad \frac{d\alpha}{dt} = -\frac{\partial H}{\partial \psi}$$

Toroidal segments (Riezlern 2005)

- space charge compensation in torodial beam transport
- investigation of beam drift effects
- investigation of beam instabillities
- experimental study of beam injection
- evaluation of numerical simulation with experimental results

Injection - Experiment





FÜR WEITERE INFOS...

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Injection – Volume Source

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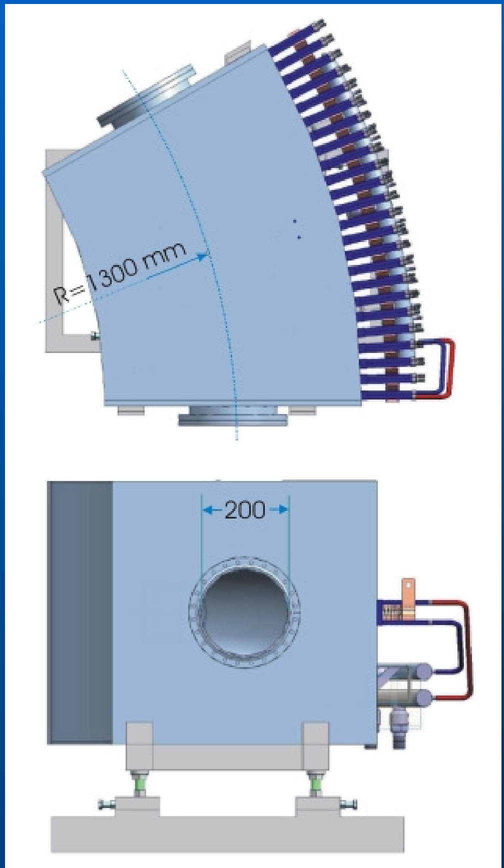
Injection – Parameters



Parameters

Toroidal Magnetic field on axis = 0.6 T

Major Radius = 1300 mm

Minor Radius = 100 mm

Injection Beam Energy = 20keV max

Injection Beam Current = 9.6 mA max

Auxiliary field (Helmholtz Coils) = 0.2 T max

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Summary

-Code developement fast finished (Testing)

-Toroidal sectors experiments this year

-Injection experiment

-Data comparison with simulations