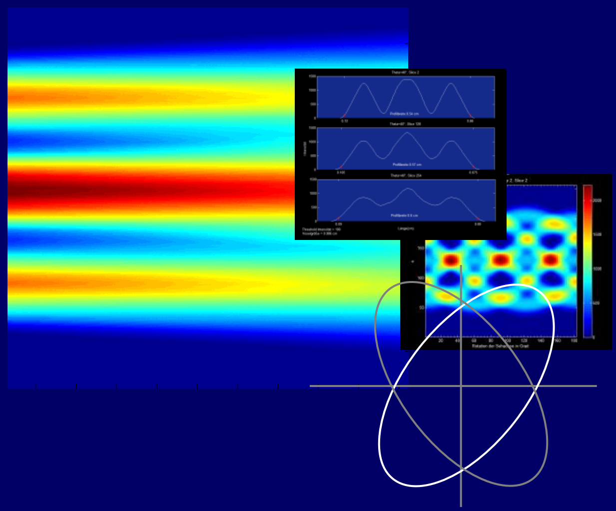
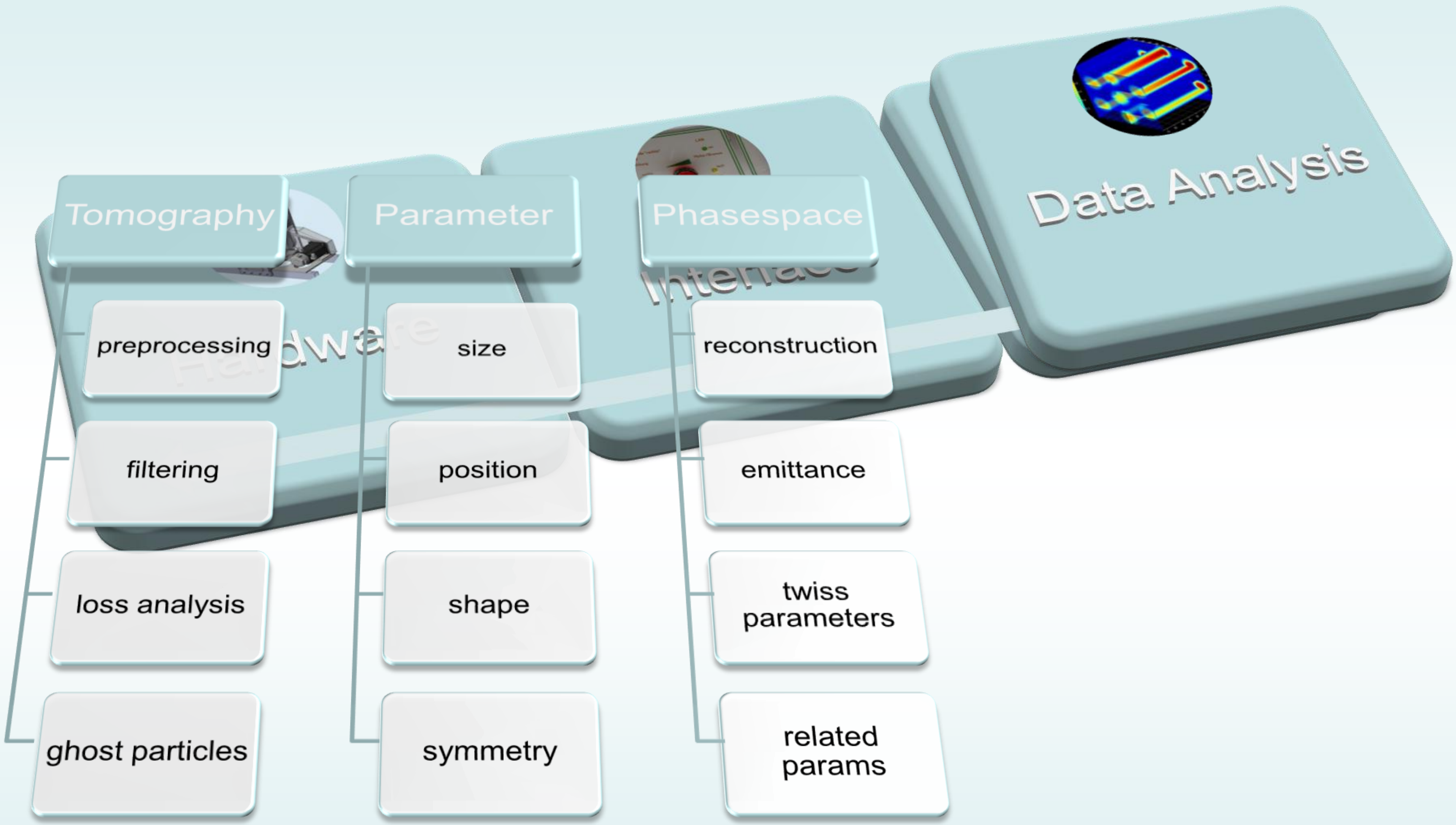
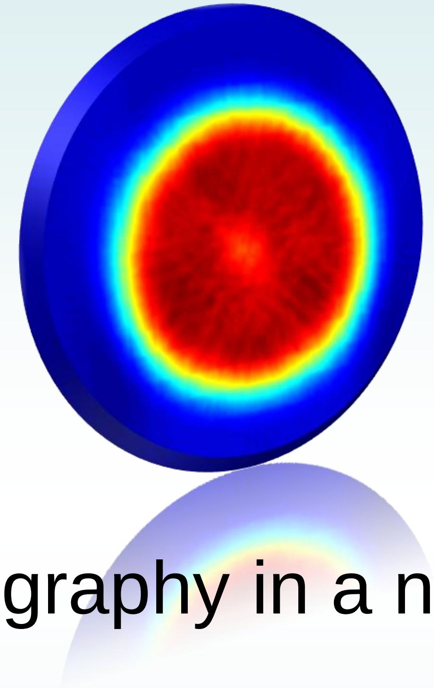


Noninvasive Optical Diagnostics for Intense Ion Beams

HICforFAIR Workshop Riezlern 2011
Institute of Applied Physics, Goethe University Frankfurt, NNP AG
Hermine Reichau

Beam Tomography for FRANZ

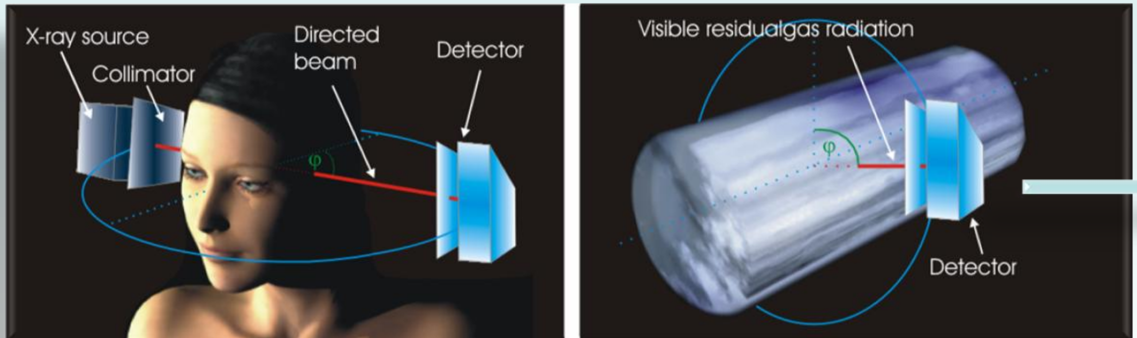


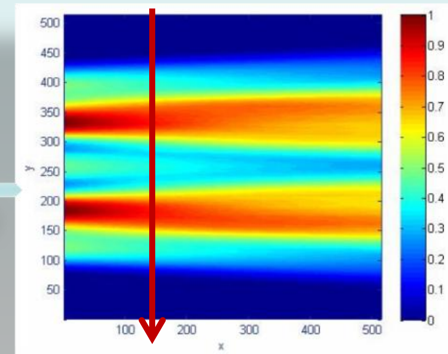


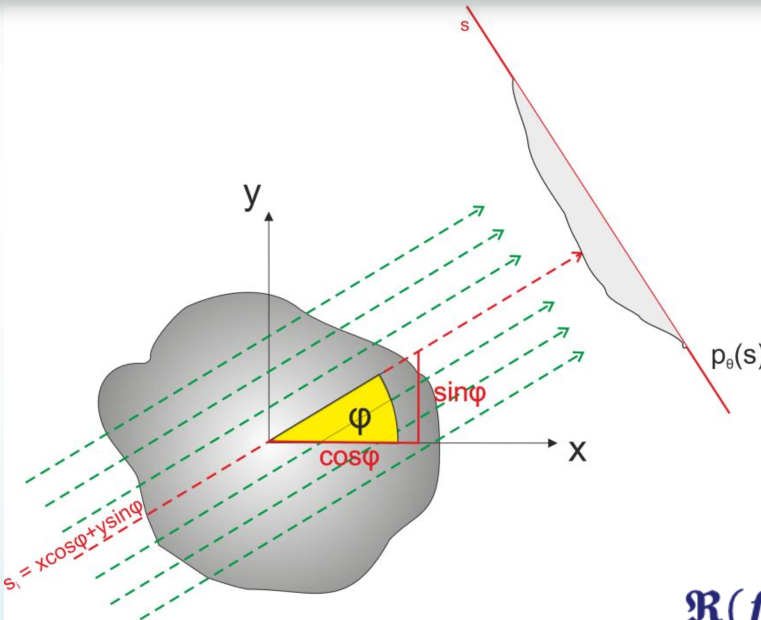
Tomography in a nutshell

Linac beams do not occure in distributions named after dead mathematicians...

Tomography







$$s = x \cdot \cos(\varphi) + y \cdot \sin(\varphi)$$

Fourier-Slice-Theorem:

$$f(x,y) \xrightarrow{\text{2DFT}} \mathcal{P}(\Theta,u)$$

$$f(x,y) \xleftarrow{\text{2D IFT}} \mathcal{P}(\Theta,u)$$

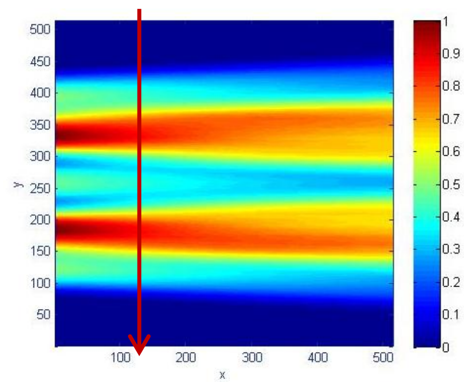
projection: $f(x,y) \downarrow p(\varphi,s)$

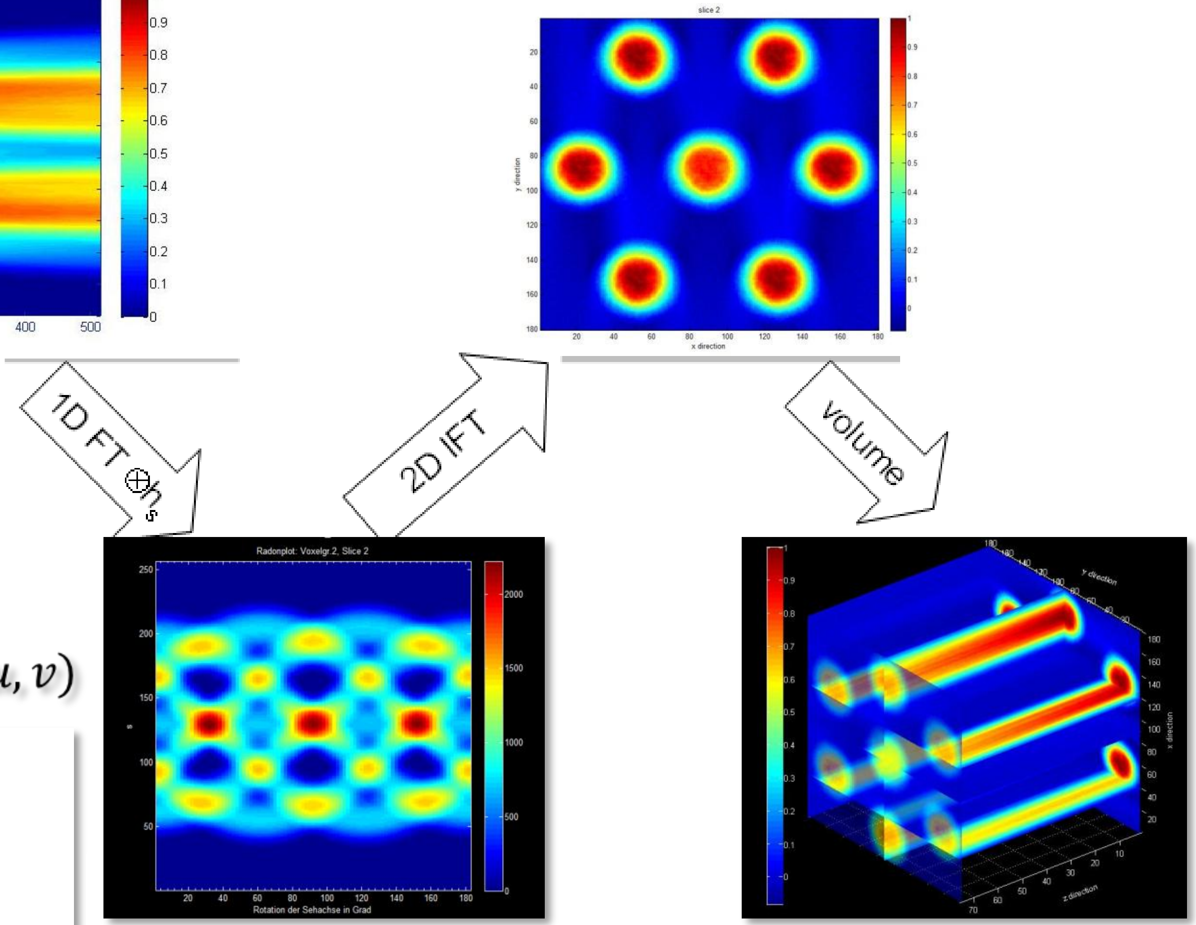
$\mathcal{P}(\Theta,u) \approx \mathcal{F}(u,v)$

$$p(\varphi,s) \xrightarrow{\text{1DFT}} \mathcal{F}(u,v)$$

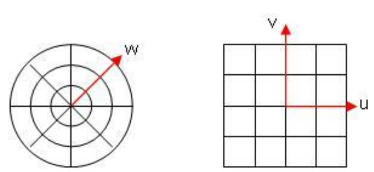
$$\boldsymbol{\Re(f)(m,b) := \int_{-\infty}^{\infty} f(x,mx+b)dx = \int_{-\infty}^{\infty} f(x,y)dx}$$

Tomography





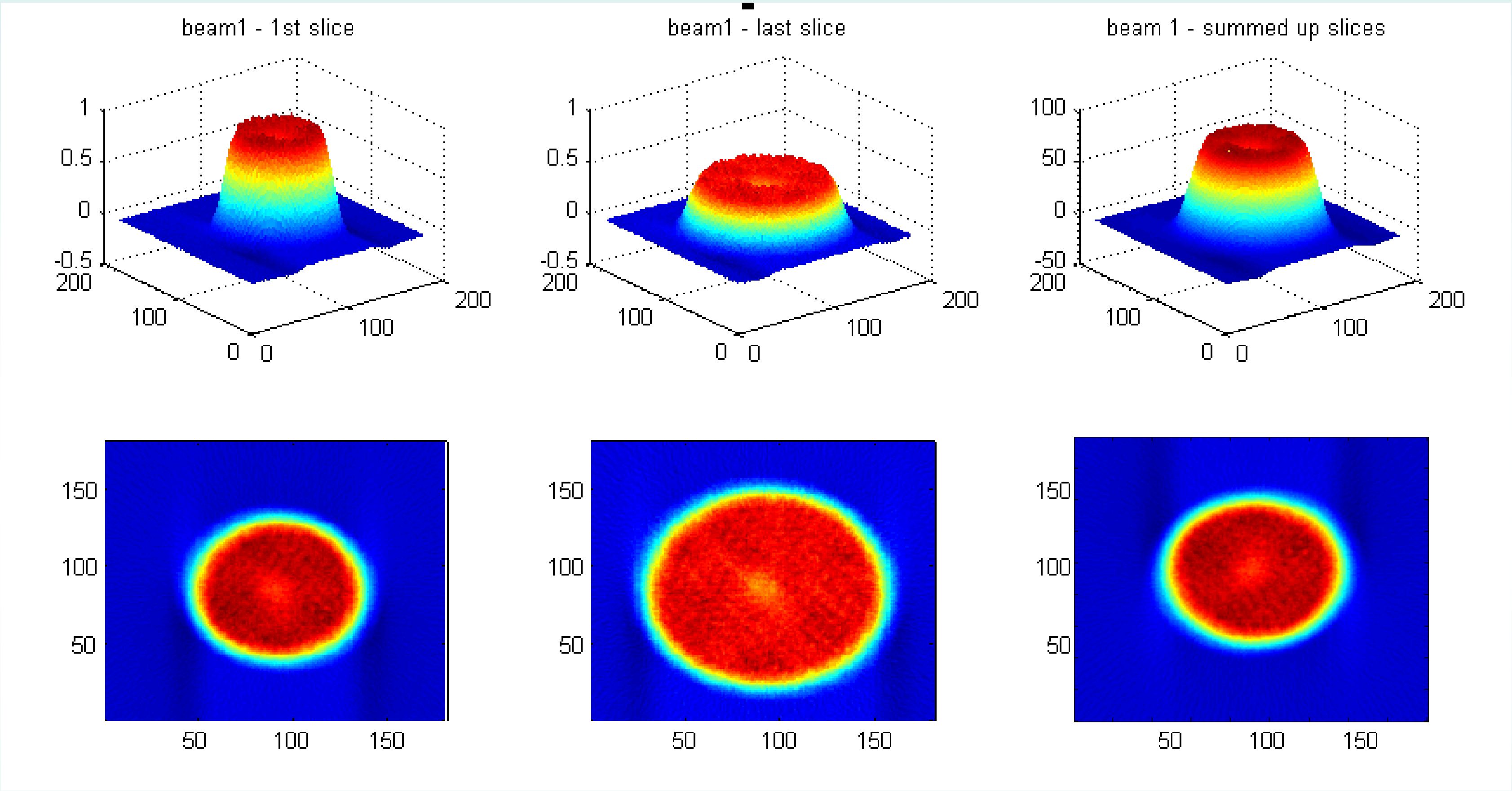
$$p\,(\varphi, s) \xrightarrow{\text{1DFT}} \mathcal{F}(u, v)$$



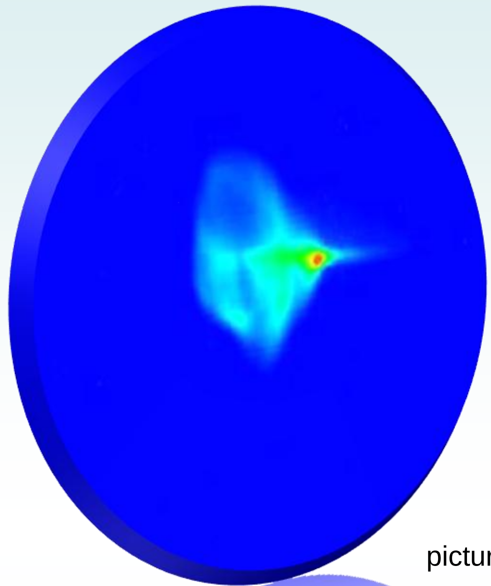
polar coordinates

cartesian coordinates

Tomography



z_1 z_n z_1-z_n



picture: P.Evtushenko(JeffersonLab)

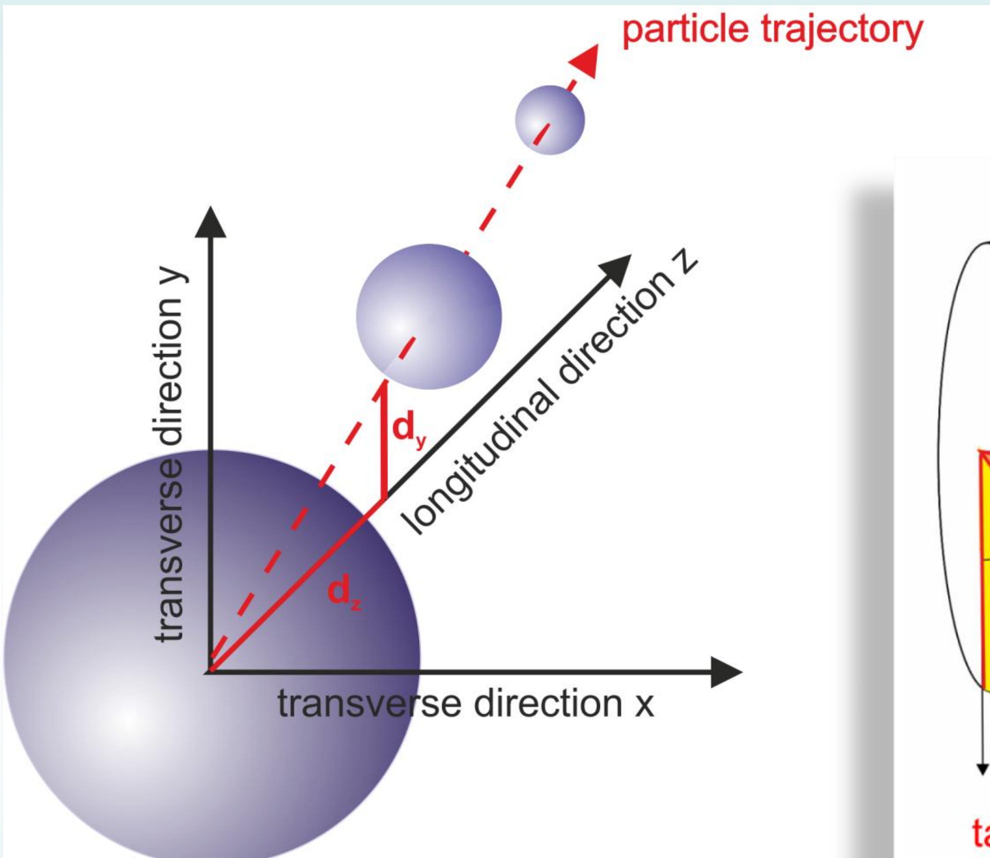
Phase Space Reconstruction

Linac beams do not occure in distributions named after dead mathematicians...

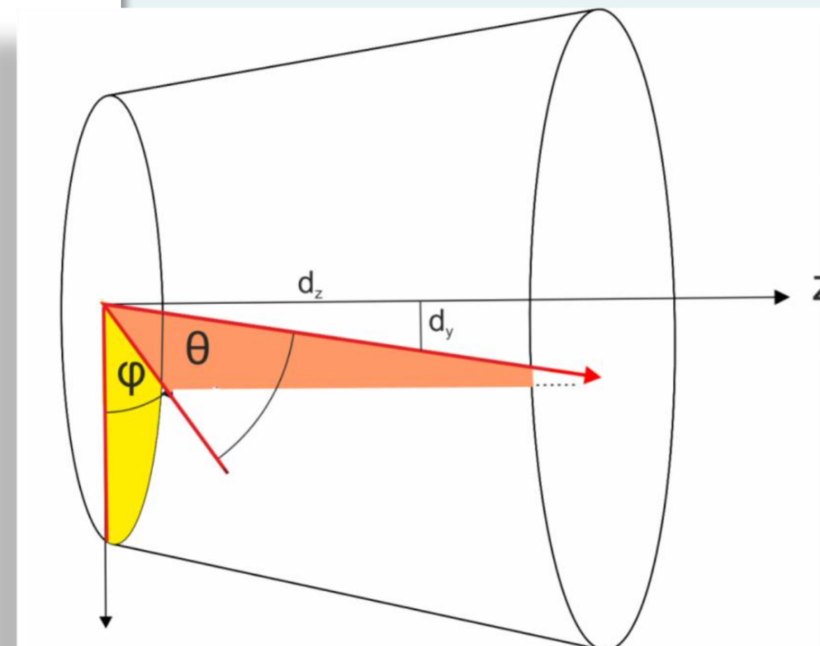
...but they look like humming birds.

P.O'Shea (University Maryland)

Phasespace Reconstruction

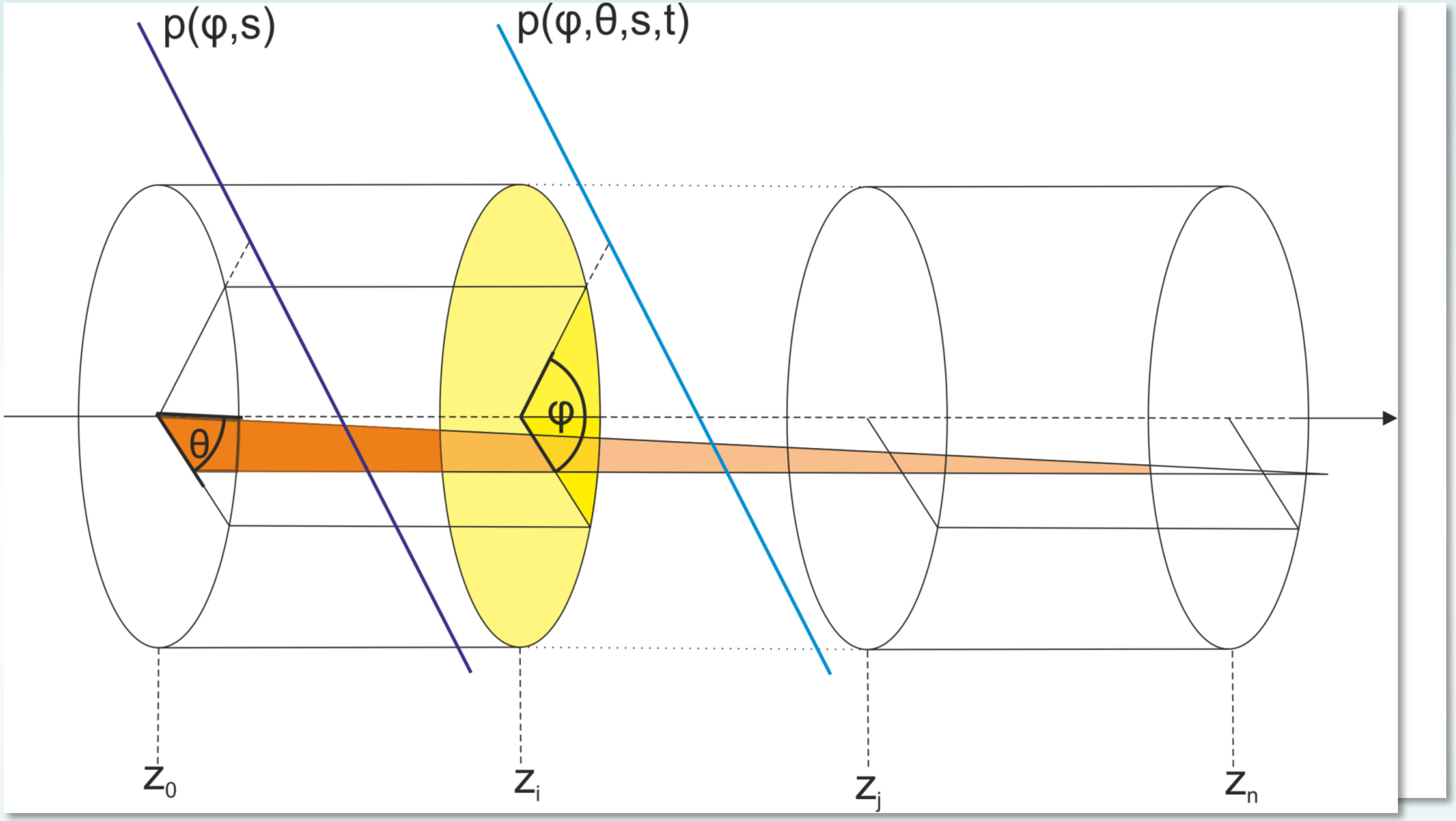


$$\frac{d_y}{d_z} = \tan x' \quad \widehat{=} \tan\theta$$



tan θ t is the slope of the projection line in z-direction, seen from angle φ

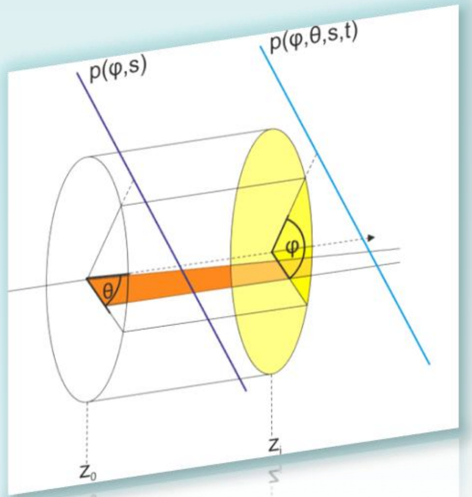
Phasespace Reconstruction



$$A_{G1} = A_D \cdot A_R$$

$$A_G = A_D \cdot A_R \cdot A_O$$

Phasespace Reconstruction



$$\begin{pmatrix} x_1 \\ x'_1 \end{pmatrix} = \mathcal{A} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$

$$p_{\varphi,\theta}{}^{s,t}(x_1) = \int \rho(x_1, x'_1)_{z_1} dx'_1$$

: = Integration over phase-space density distribution along x'_1 is equivalent to the beamprofile along x_1

$$p_{\varphi,\theta}{}^{s,t}(x_1) = \int\int \rho(x_1, x'_1)_{z_1} \delta(x_1 - x) dx_1 dx'_1$$

Liouville: particle densitiy distribution does not change:

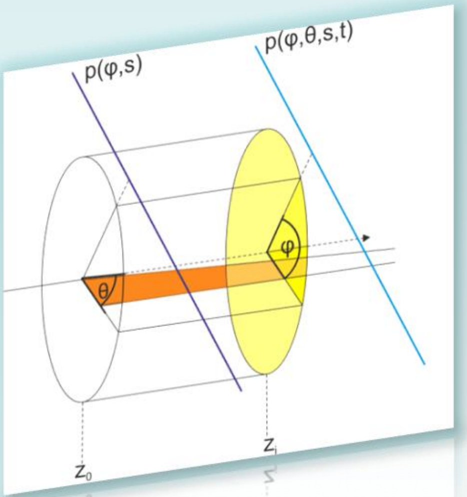
$$\rho(x_0, x'_0)_{z_0} = \rho(x_1, x'_1)_{z_1}$$

$$p_{\varphi,\theta}{}^{s,t}(x) = \int\int \rho(x_0, x'_0)_{z_0} dx'_0 \delta(\mathcal{A}_{11} x_0 + \mathcal{A}_{12} x'_0 - x) dx_0 dx'_0$$

scaling factor: $t = \sqrt{\mathcal{A}_{11}{}^2 + \mathcal{A}_{12}{}^2}$ phase space rotation angle: $\tan\theta = \frac{\mathcal{A}_{12}}{\mathcal{A}_{11}}$

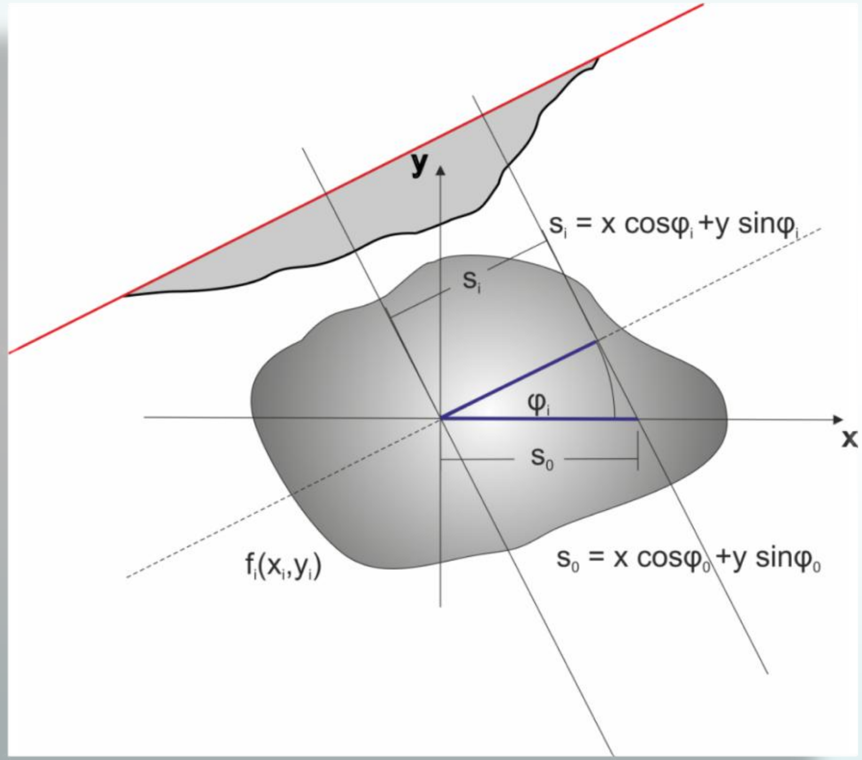
$$p_{\varphi}{}^{s,t}(\theta) = \frac{1}{t} \iint \rho(x_0, x'_0)_{z_0} \delta(x_0 \cdot \cos\varphi + x'_0 \cdot \sin\varphi - \frac{x}{t}) dx_0 dx'_0$$

Phasespace Reconstruction

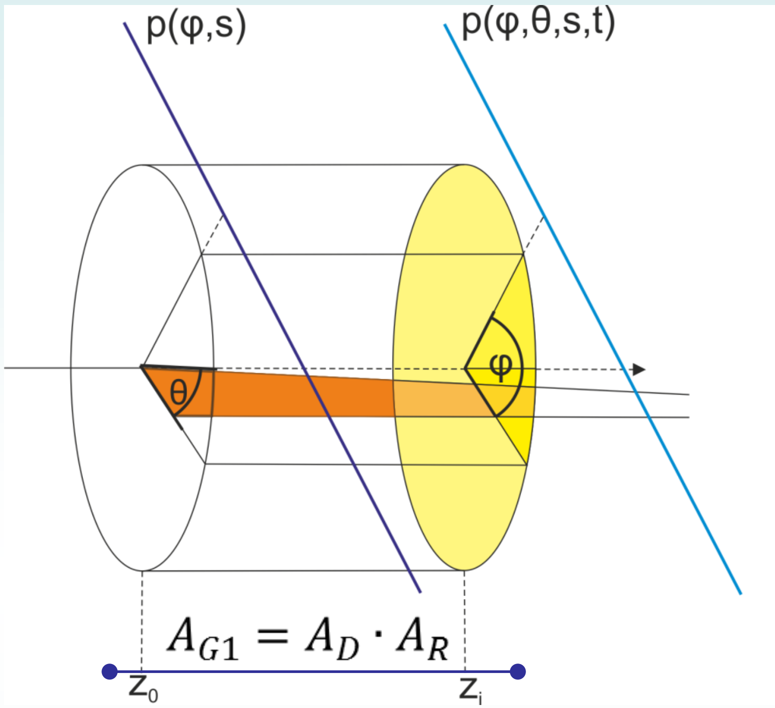


$$p_{\varphi}{}^{s,t}(\theta) = \frac{1}{t}\iint \rho(x_0, x'_0)_{z_0}\delta(x_0\cdot\cos\varphi + x'_0\cdot\sin\varphi - \frac{x}{t})dx_0dx'_0$$

$$\tilde{f}\left(\frac{x}{t},\theta\right) = t\cdot p_{\varphi}{}^{s,t}(x,\theta)$$



Phasespace Reconstruction

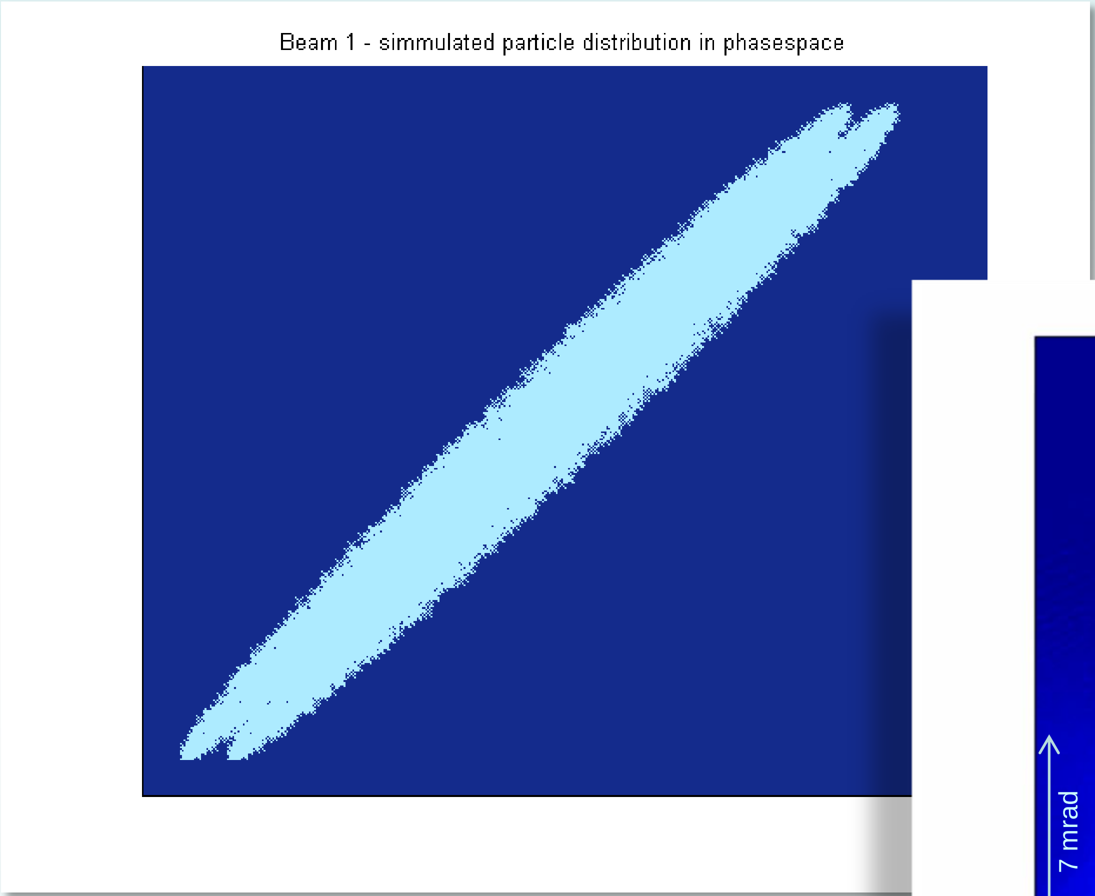


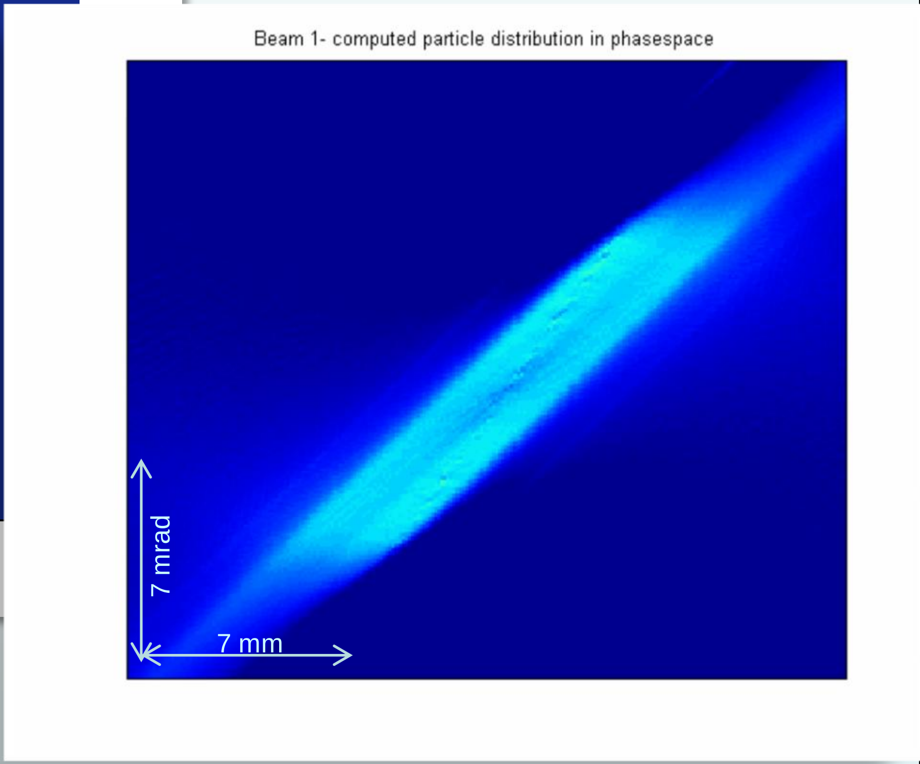
$$A_{G1} = A_D \cdot A_R$$
$$= \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}$$
$$= \begin{pmatrix} \cos\varphi - d \cdot \sin\varphi & \sin\varphi + d \cdot \cos\varphi \\ 0 & \sin\varphi + \cos\varphi \end{pmatrix}$$

$$t = \sqrt{\mathcal{A}_{11}{}^2 + \mathcal{A}_{12}{}^2} = 1 + d^2$$

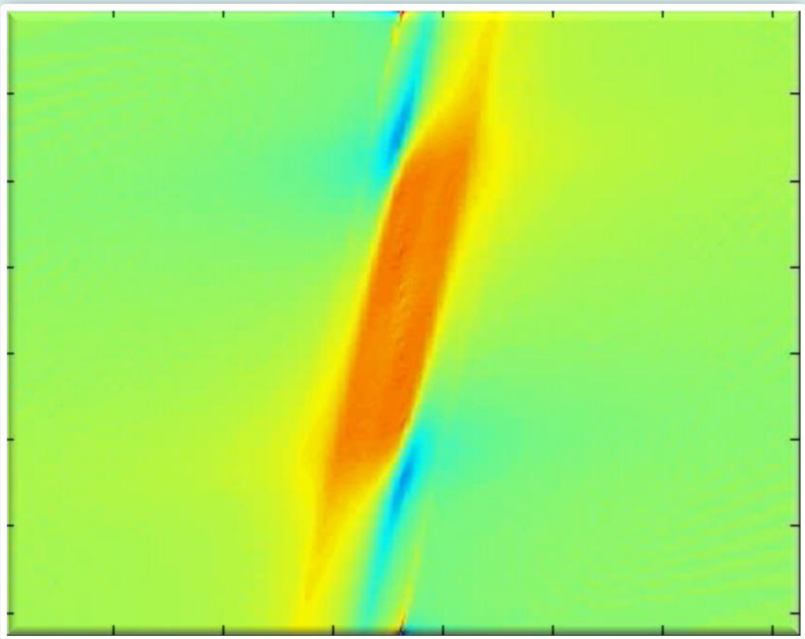
$$\theta = atan\left(\frac{\sin\varphi + d \cdot \cos\varphi}{\cos\varphi - d \cdot \sin\varphi}\right)$$

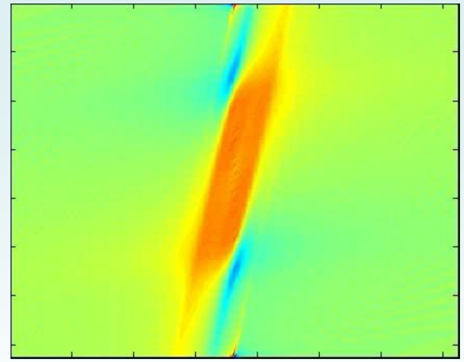
Phasespace Reconstruction

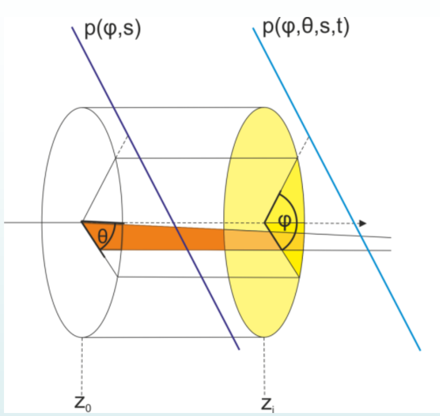


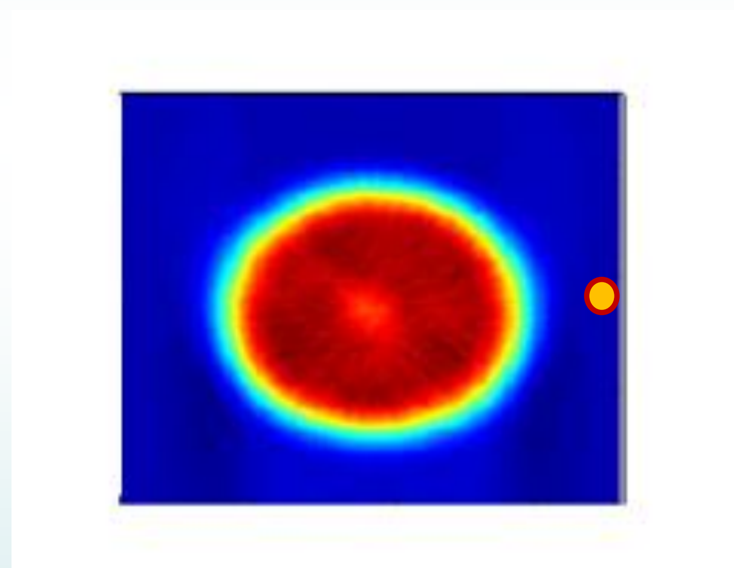


Phasespace Reconstruction

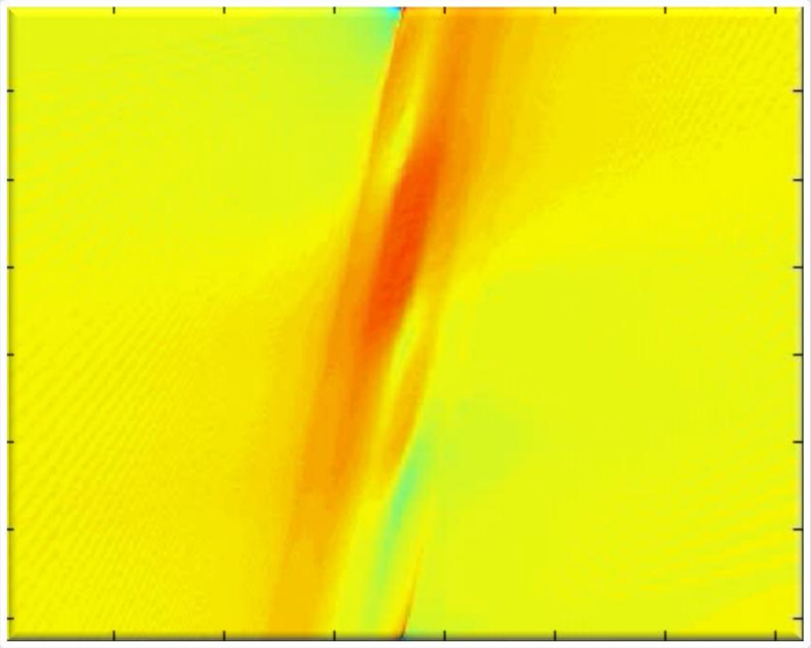


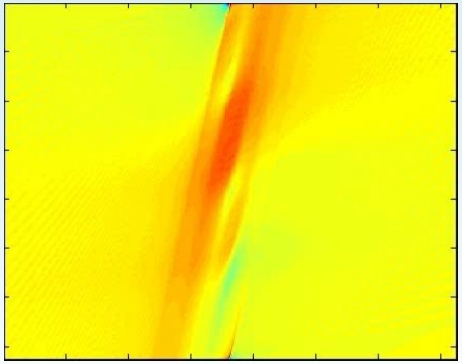


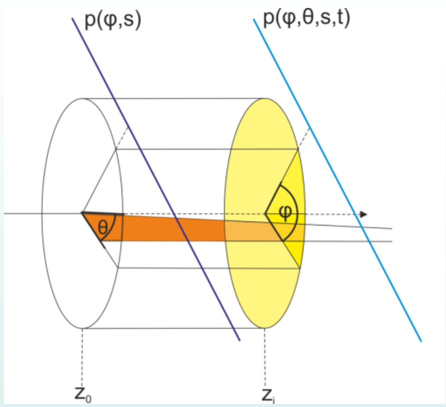


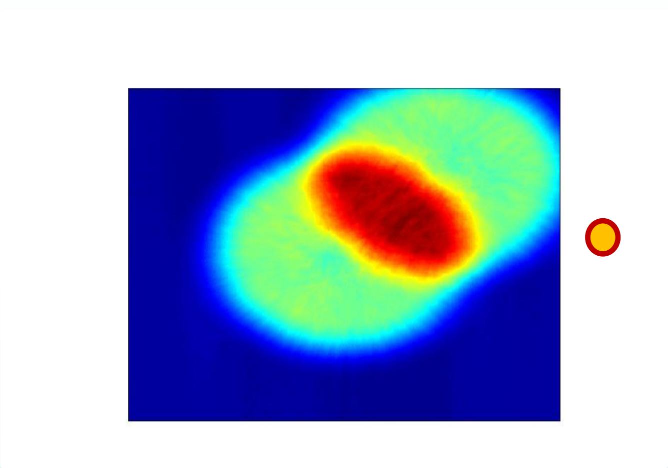


Phasespace Reconstruction

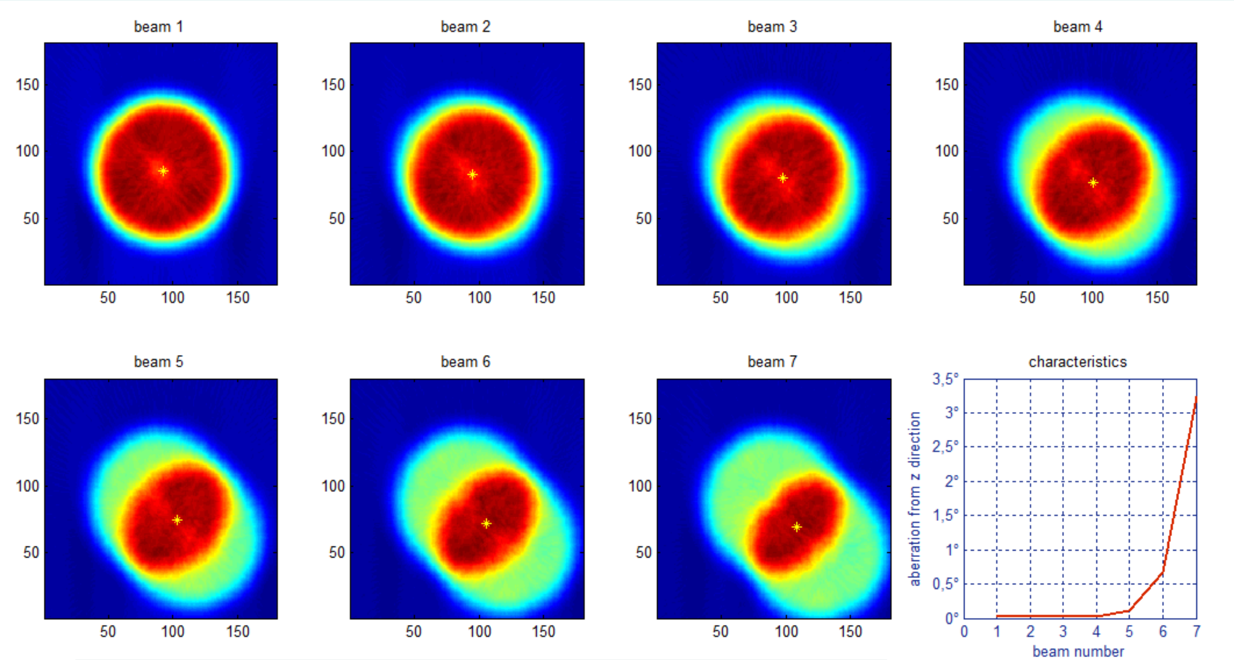


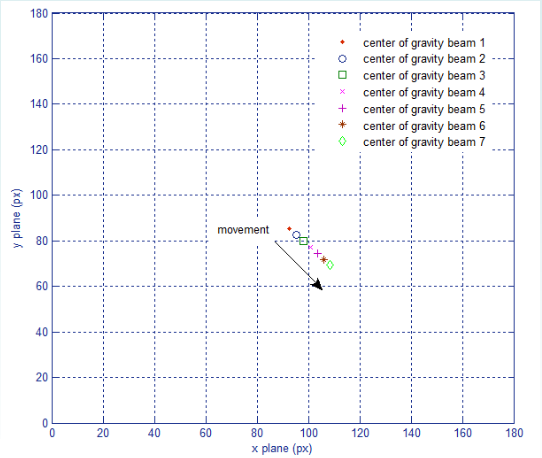


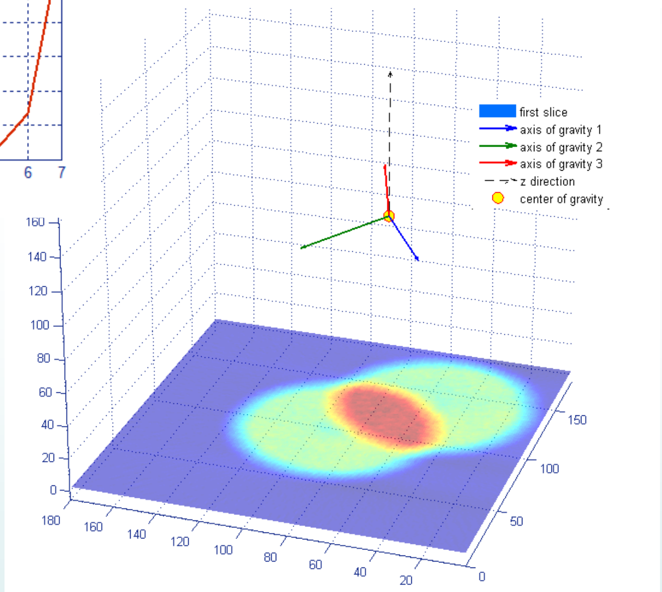


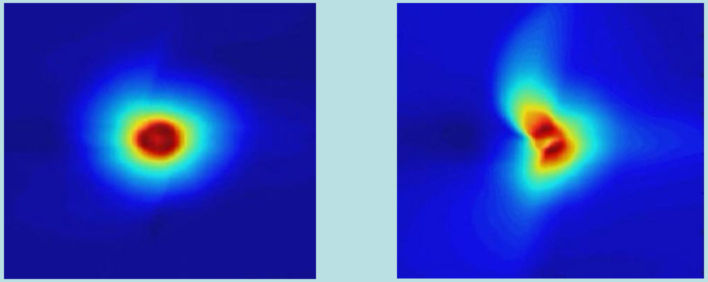


Phasespace Reconstruction









longitudinal impulses/transversal impulses
in xy-plane

Thank you for your attention!