Numerical Models for the Investigation of Charged Particle Motion

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Motivation
Symplectic Integrator
Space-charge (PIC)
Collective phenomena
Motivation

- Tracking Programs – 6D
  - paraxial Approximation
  - higher order maps
  - time integration – example

- PIC-Methods -> efficiency, parallelism
- Binary collisions -> plasma simulations
- Secondary particles -> electron clouds
Particle motion

- Particle motion – time evolution
  \[
  \dot{z} = \{z, H\}
  \]

- Formal
  \[
  \dot{z} = D_H z
  \]

- Conservation of two-form (bilinear form) = symplectomorphismus (isomorphismus of Symplectic manifolds)
  \[
  \{f, g\} = \sum_{i=1}^{N} \left[ \frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i} \right]
  \]
  \[
  z(\tau) = \exp(\tau \cdot D_H) \cdot z(0)
  \]
  \[
  dp \wedge dq
  \]
Numerical

- **Formal**
  \[ \dot{z} = D_H z \]

Looking for numerical scheme (operator $D_H$, symplectic integrator that also conserves two-form) \(\Rightarrow\) matrix

\[ M^T \Omega M = \Omega \]

\[ \Omega = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix} \]

- **Example**

\[ H(p, q) = T(p) + V(q) \]

- **Explicit and Implicit schemes**

\[ \frac{\ddot{p}^{i+1} - \dot{p}^i}{dt} = qE^i + q \frac{\dot{p}^i + \ddot{p}^{i+1}}{2m} \times B^i \]

\[ \frac{\ddot{q}^{i+1} - \dot{q}^i}{dt} = \frac{\dddot{p}^{i+1} + \dddot{p}^i}{2m} \]
Examples

Mirror configuration

Injection system
- Proton beam
- Toroidal magnetic field

Focusing on sc-toroid
- R = 1 m
- B = 8 T
- p, W = 200 MeV
- 12 beamlets

FRC-Field reversed configuration
- Fusion, targets

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Toroidal beam transport

- Low energy (10keV) composited ion beam
- The separation between species due to curvature drift possible over long path length
- Separation due to phase difference in Larmor gyration
Multispecies – Beam collimation
Secondary electrons – -> electron clouds
Two stream instability
NNP
Beam plasma interaction

Particle in cell Method
Important
- computation step \( dt \)
- mesh size
Target Normal Sheath Acceleration (TNSA)

- Focusing (Pulsed Solenoid ~ 18T)
- Injection and Post-acceleration in CH-Structure

Simulation – Protons & Electrons

\[ dt = 25 \text{fs} \]

R = 30 \text{\mu m}, \ L = 22 \text{\mu m}

Protons \ W = 10 \text{MeV}
Electrons \ W = 5.5 \text{keV}

Particles/1 Macroparticle = 4444 \Rightarrow 4.5 \text{Mio}

Macroparticles

Mesh:
\[ \text{dr} = 6 \text{\mu m} \]
\[ \text{d} \phi = 0.2 \text{rad} \]
\[ \text{dz} = 2 \text{\mu m} \]
Simulation - Improvements

- \( dt = 5 \text{fs} \)
- \( R = 30 \mu m, \ L = 22 \mu m \)
- Protons \( W = 10 \text{MeV} \)
- Electrons \( W = 5.5 \text{keV} \)
- Particles/1 Macroparticle = 4444 \( \Rightarrow \) 4.5Mio Macroparticles
- Mesh:
  - \( dr = 6 \mu m \)
  - \( d\phi = 0.2 \text{rad} \)
  - \( dz = 2 \mu m \)

Less separation
Lower electric fields and potential

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Plasma oscillation longitudinally
Along magnetic field

Due to the higher magnetic field in propagation direction
Redistribution of longitudinal momentum
To the transverse direction
Ratio – variation of total energy/energy (~ 1e-5 @ 3ps)

- Less comparing with previous case 6%
- Due to the variation of magnetic field?
- Using different type of integrators
- Longer simulation needed

- Cyclotron frequency -> characteristic time $\tau_c = 6e-12s$
- Plasma frequency -> characteristic time $\tau = 3.5e-13s$
- Debye length $\lambda_D = 0.2\mu m$

New strategy -> finer mesh dual mesh
Dynamic in strong magnetic field (Solenoids, Toroids & Fringing fields + magnetic coupling)
Collective phenomena (Gabor Lens, electrons & ions)
Correction coils
Space-charge effects & Aberrations
Experience -> Development of efficient simulation and design tools for Accelerators