Time Reversal in Beam Dynamics

Test with LORASR

Long Phi Chau

Outline:

- Motivation
- Mathematical meaning of time reversal
- Conclusion for beam dynamics
- Test with LORASR

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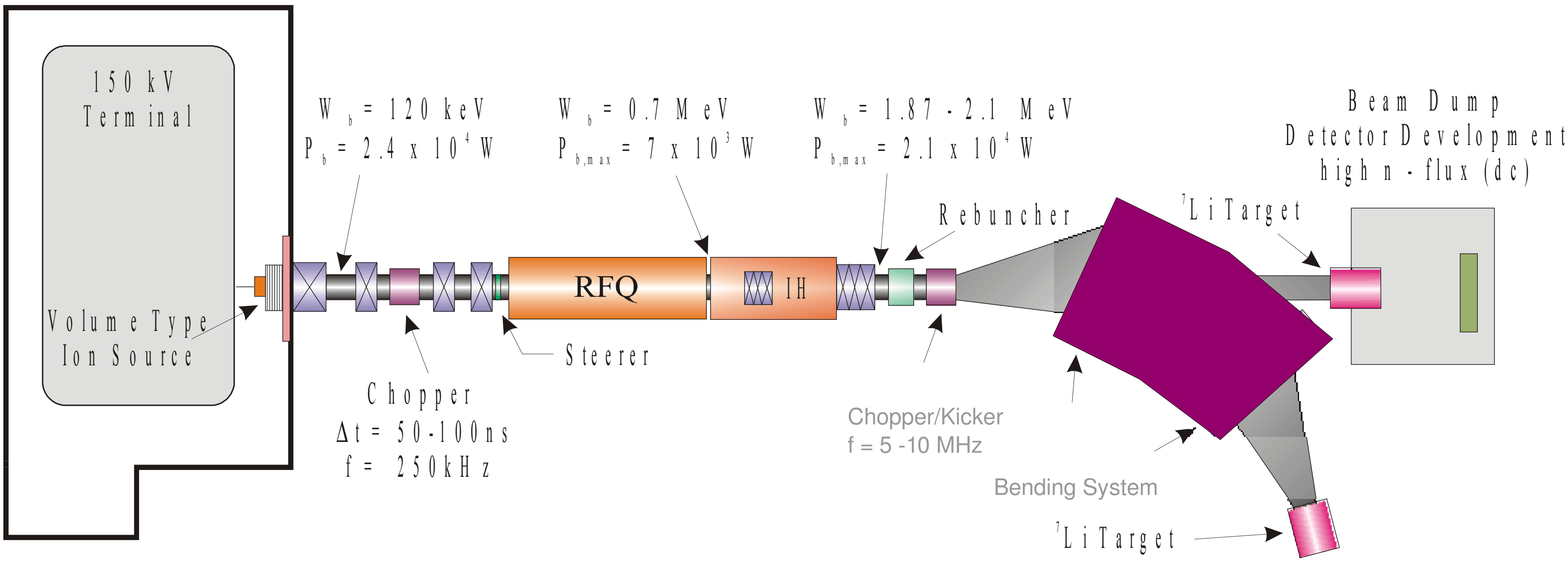
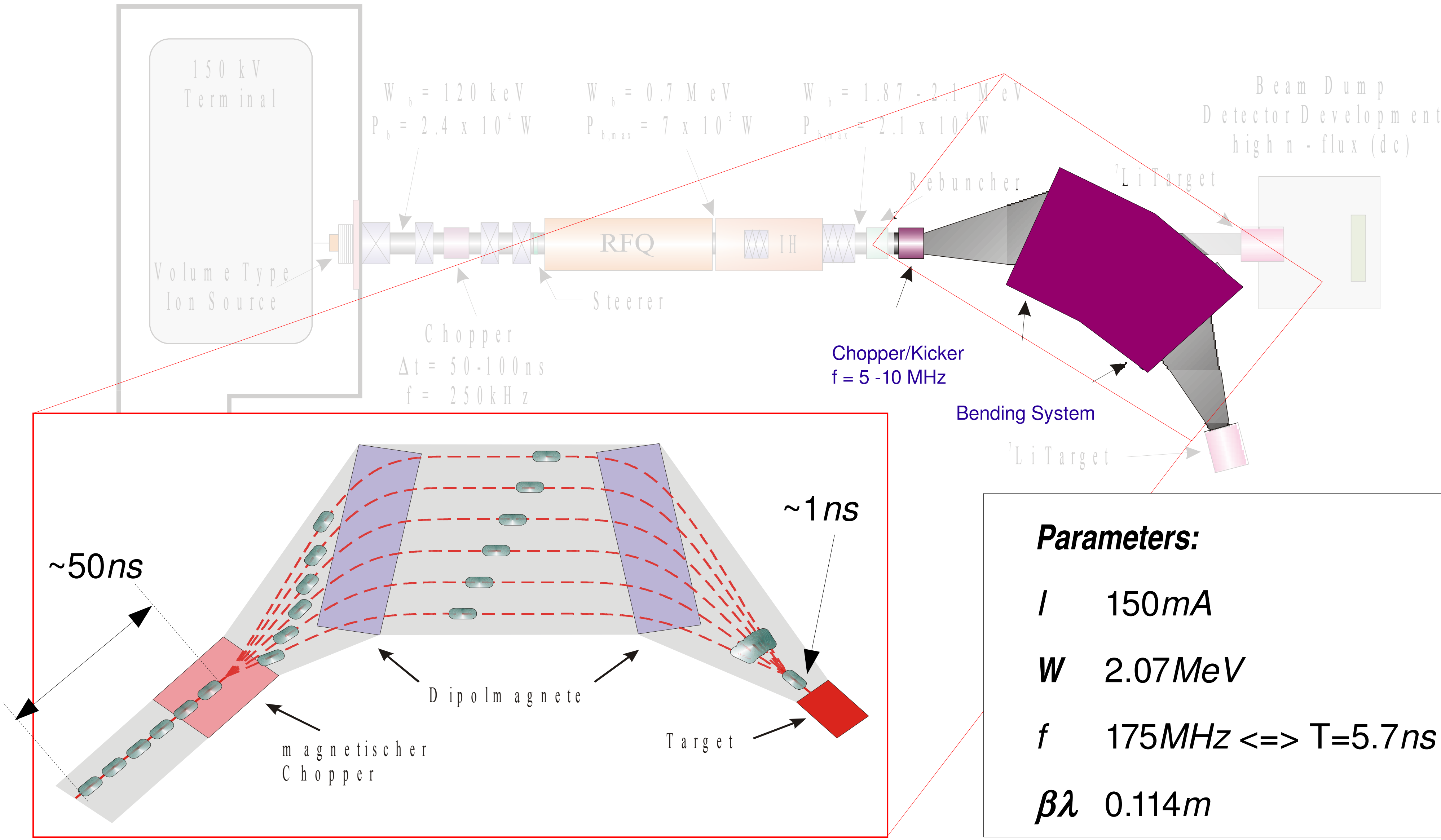
Schematic Layout of FRANZ



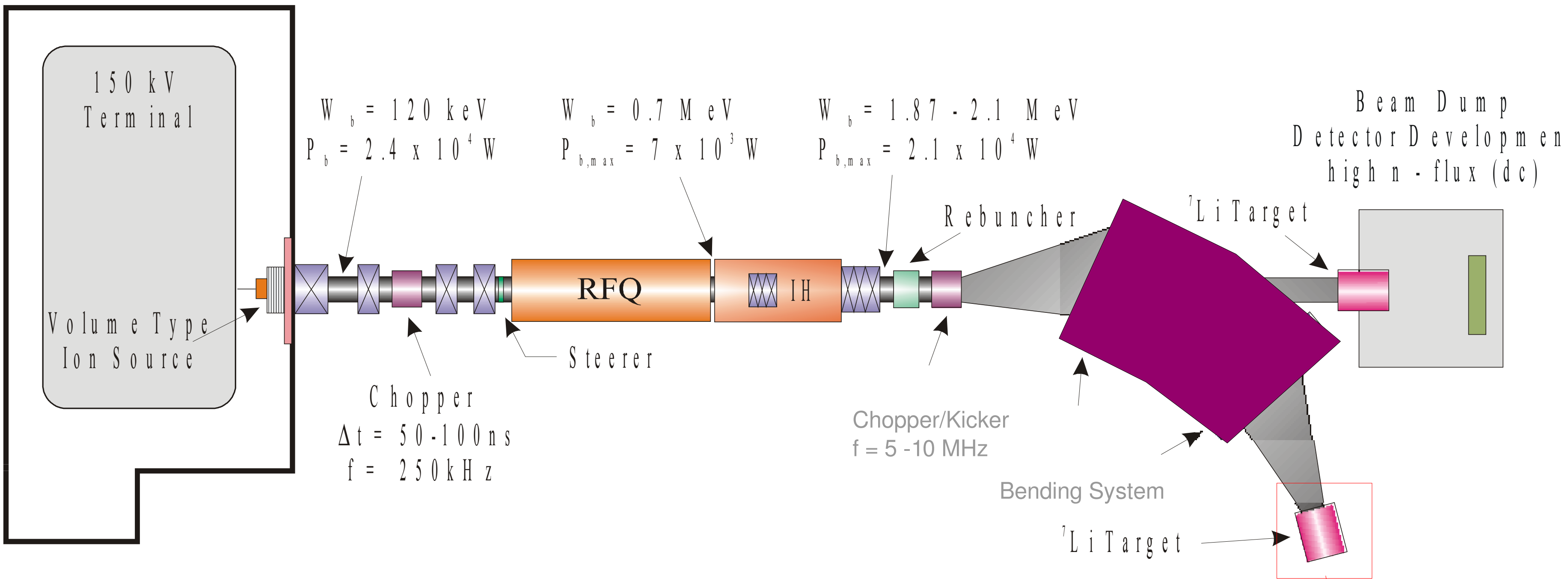
One *nanosecond* bunch compressor for high intense proton beam



Parameters:

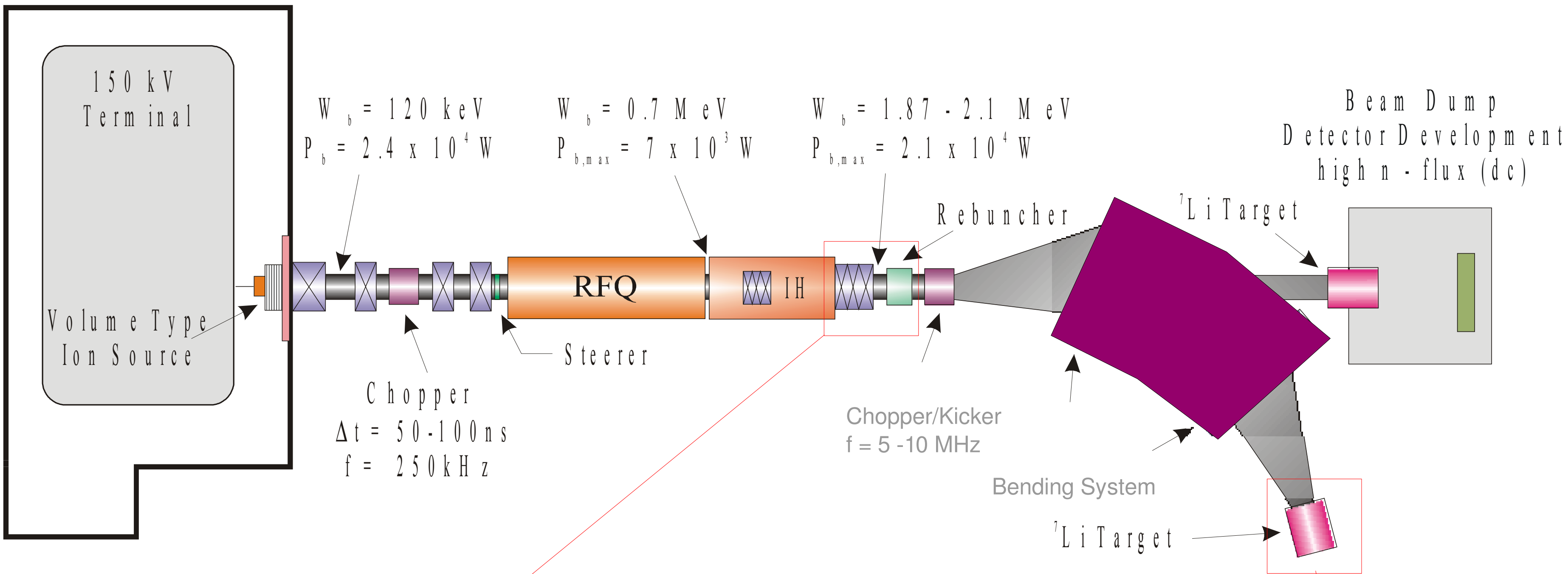
I	150 mA
W	2.07 MeV
f	175 MHz <=> T=5.7 ns
$\beta\lambda$	0.114 m

One *nanosecond* bunch compressor for high intense proton beam



Output distribution is given by the requirement of the experiments

One *nanosecond* bunch compressor for high intense proton beam



Quadropole triplet + Rebuncher cavity:

=> degree of freedom for input distribution of the bunch compressor.

Output distribution is given by the requirement of the experiments

Time Reversal in Beam Dynamics

Motivation:

- Estimation of ***acceptance*** for a given distribution at the target
- Design a bunch compressor ***without rebuncher possible*** **(?)**

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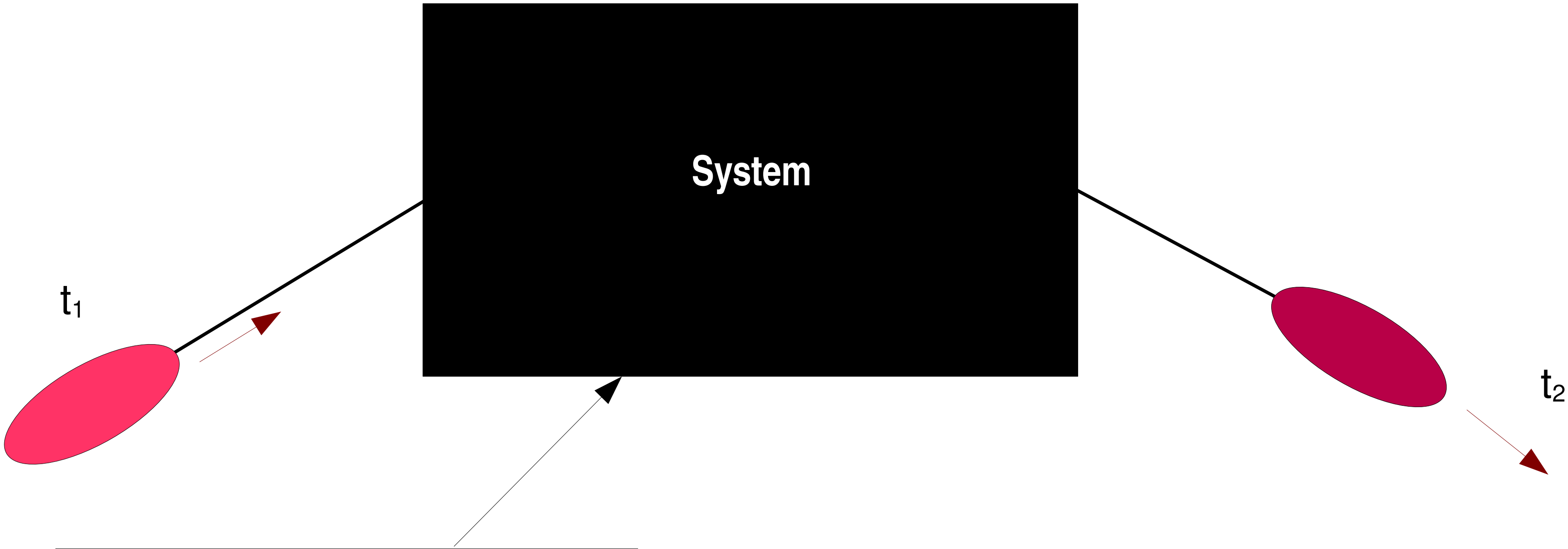
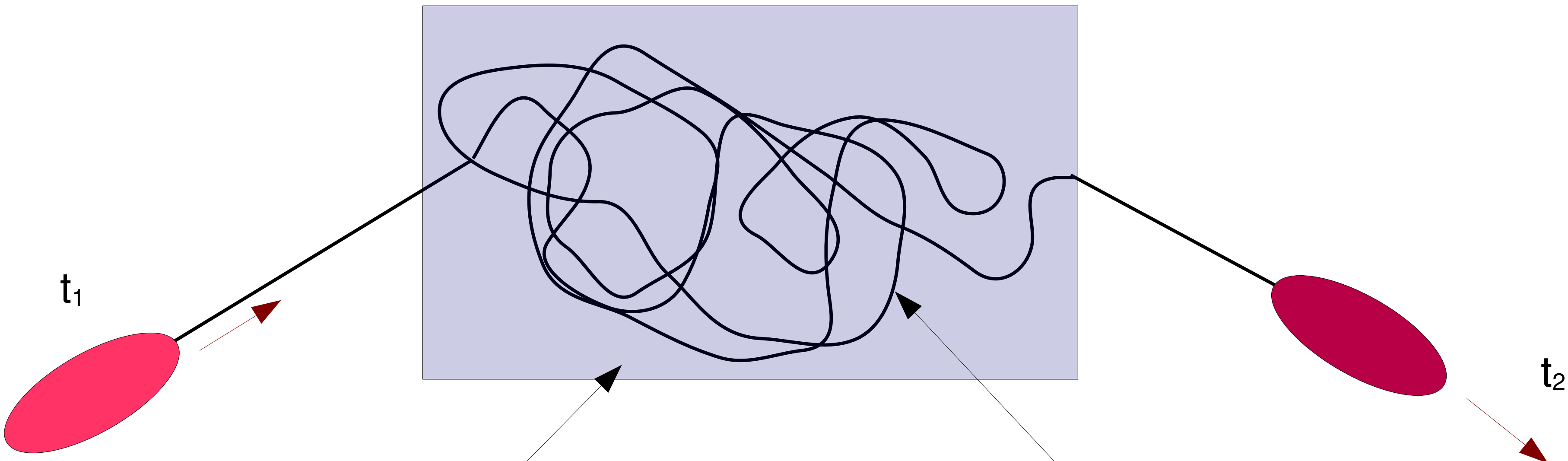
Mathematical Meaning of Time Reversal



Assumptions:
- reversible interaction
<=> no dissipation
- ***elastic scattering***

Mathematical Meaning of Time Reversal



Assumptions:
- reversible interaction
<=> no dissipation
- ***elastic scattering***

=>

Implication:
well-defined trajectory r(t)

Mathematical Meaning of Time Reversal

- ***Trajectories r(t): well-defined*** for given initial conditions
- Time t - > arbitary parameter
- ***Reparameterization*** possible: e.g. **T: t -> λ = -t**

=> | ***mathematical*** **time reversal <=> reparameterization** |

$$\vec{r}\,(t) \;=\; \vec{r}\,(\lambda) \qquad \text{with} \quad \lambda \;=\; -t$$

Mathematical Meaning of Time Reversal

Velocity:

$$\vec{v}(\lambda) = \frac{d\vec{r}(t(\lambda))}{d\lambda} = \underbrace{\frac{d\vec{r}(t)}{dt}}_{\vec{v}(t)} \cdot \underbrace{\frac{dt(\lambda)}{d\lambda}}_{-1} \qquad \text{with} \quad \lambda = -t$$

$$\Rightarrow \boxed{\vec{v}(-t) = -\vec{v}(t)}$$

Acceleration:

$$\vec{a}(\lambda) = \frac{d}{d\lambda}\vec{v}(\lambda) = \frac{d}{d\lambda}\Big(\vec{v}(t) \cdot \underbrace{\frac{dt(\lambda)}{d\lambda}}_{-1}\Big) = -\frac{d}{d\lambda}\frac{d\vec{r}}{dt} = -\frac{d}{dt}\frac{d\vec{r}}{d\lambda} = -\frac{d}{dt}(-\vec{v}(t)) = \vec{a}(t)$$

$$\Rightarrow \boxed{\vec{a}(-t) = \vec{a}(t)}$$

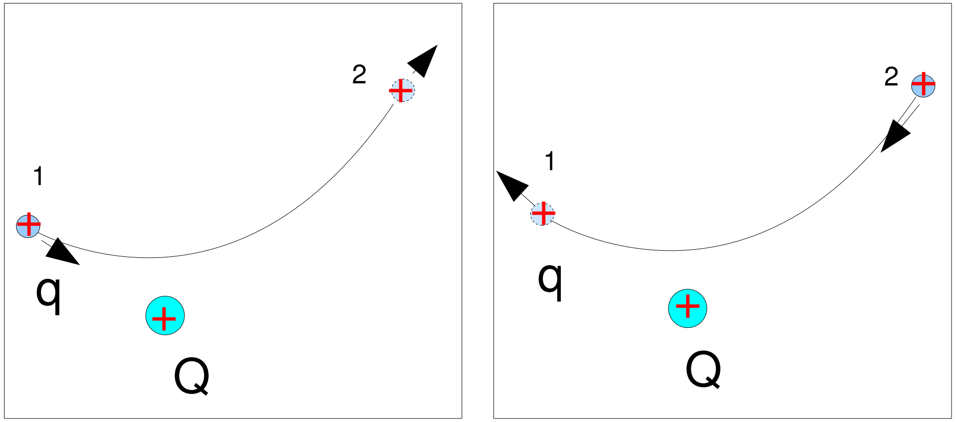
=>

Momentum: $\vec{p}(-t) = -\vec{p}(t)$

Force: $\vec{F}(-t) = \vec{F}(t)$

Mathematical Meaning of Time Reversal: „E- , B-Field“

Laboratory frame (!):



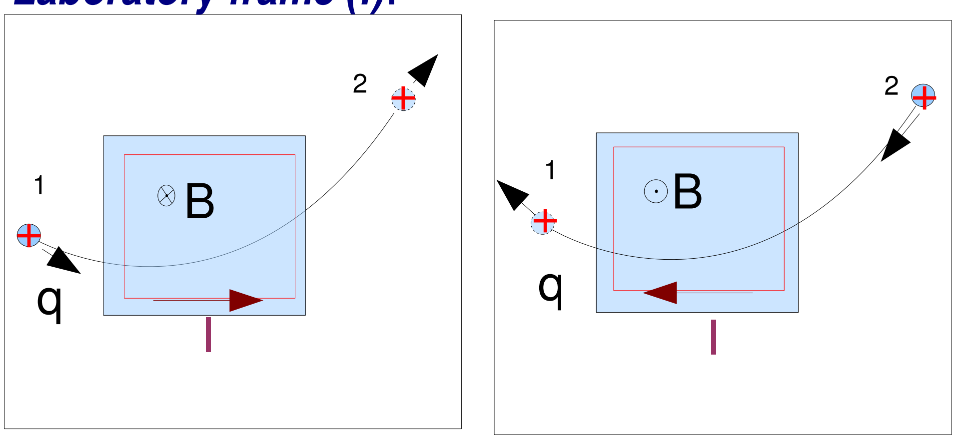
$$\vec{F}(-t) = \vec{F}(t)$$
$$q\,\vec{E}(-t) = q\,\vec{E}(t)$$

$$\Rightarrow \quad \boxed{\vec{E}(-t) = \vec{E}(t)}$$

=> ***Electric field: invariant***

$$\boxed{T: \quad t \rightarrow \lambda = -t}$$

Laboratory frame (!):



$$\vec{F}(-t) = \vec{F}(t)$$
$$q(\vec{v}(-t)\times\vec{B}(-t)) = q(\vec{v}(t)\times\vec{B}(t))$$
$$q(-\vec{v}(t)\times\vec{B}(-t)) = q(\vec{v}(t)\times\vec{B}(t))$$

$$\Rightarrow \quad \boxed{\vec{B}(-t) = -\vec{B}(t)}$$

=> ***Magnetic field: reversed***

<=> Reversal of the current <=> $v(-t)= -v(t)$

Mathematical Meaning of Time Reversal

Mathematical time reversal: $T: \quad t \quad \rightarrow \quad \lambda = -t$

$$\begin{pmatrix} \vec{r}(t) \\ \vec{v}(t) \\ \vec{E}(t) \\ \vec{B}(t) \end{pmatrix} \xrightarrow{\quad T \quad} \begin{pmatrix} \vec{r}(t) \\ -\vec{v}(t) \\ \vec{E}(t) \\ -\vec{B}(t) \end{pmatrix}$$

Mathematical Meaning of Time Reversal

Mathematical time reversal: $T: \quad t \quad \rightarrow \quad \lambda = -t$

$$\begin{pmatrix} \vec{r}(t) \\ \vec{v}(t) \\ \vec{E}(t) \\ \vec{B}(t) \end{pmatrix} \xrightarrow{\;T\;} \begin{pmatrix} \vec{r}(t) \\ -\vec{v}(t) \\ \vec{E}(t) \\ -\vec{B}(t) \end{pmatrix}$$

Note: „mathematical time revesrsal“ $\neq$ turn back a „real“ system to its initial state

=> there ***should*** be no increasing or decreasing of physical enthropy for the ***discussed system***!

(reversible interaction)

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- Conclusion for beam dynamics
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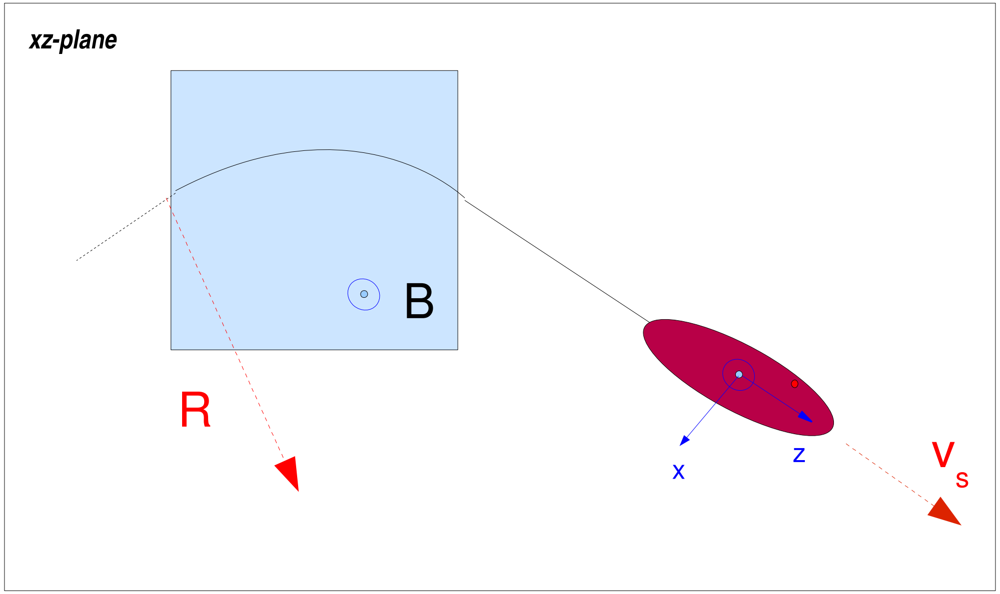
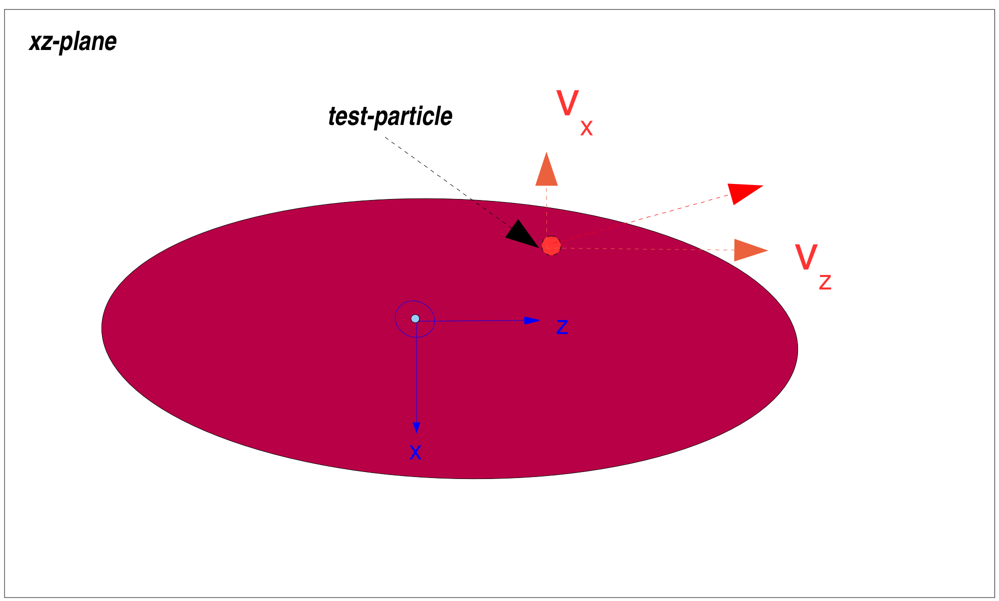
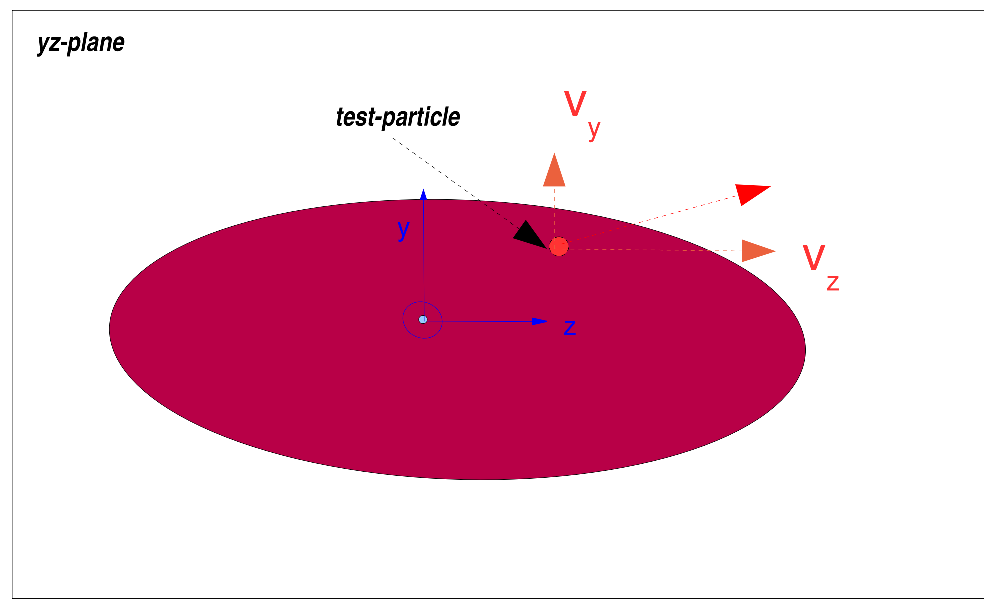
Conclusion for Beam Dynamics: „Moving Frame“



conventions:

- (x,y,z)-> left handed, moving frame
- z -> direction of the ***center particle***
- x -> direction of the ***radius of curvature***





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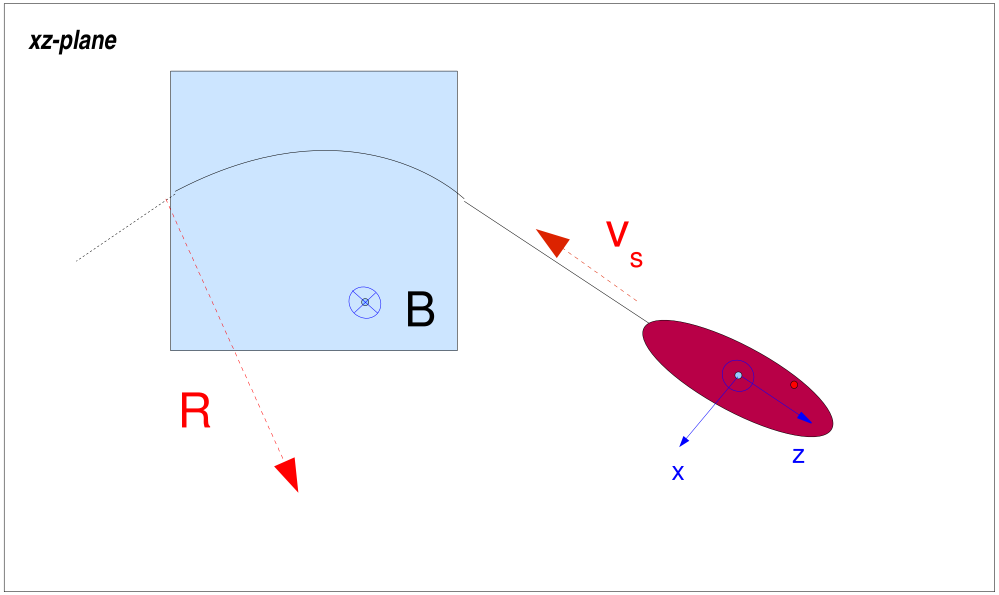
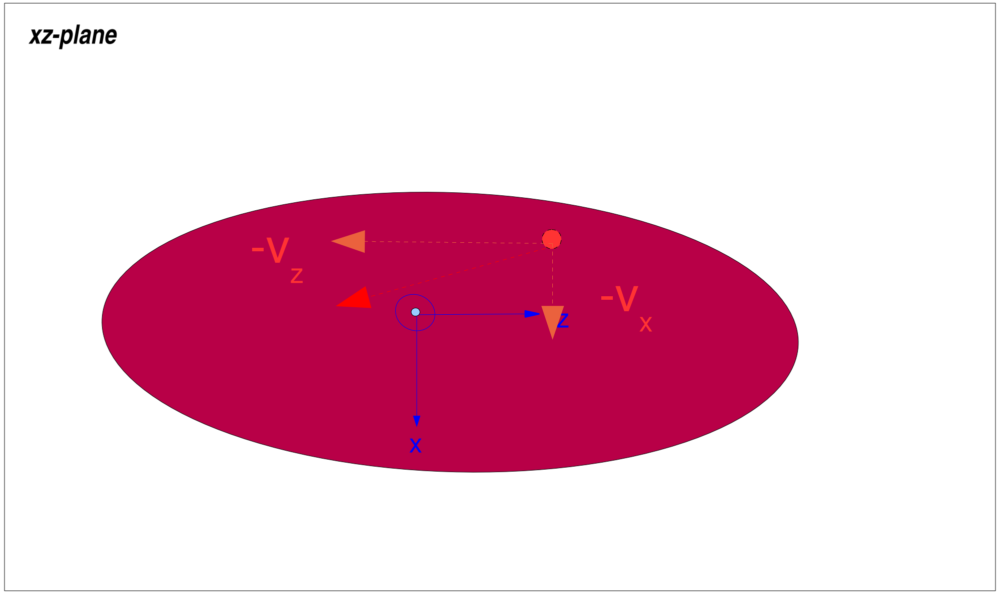
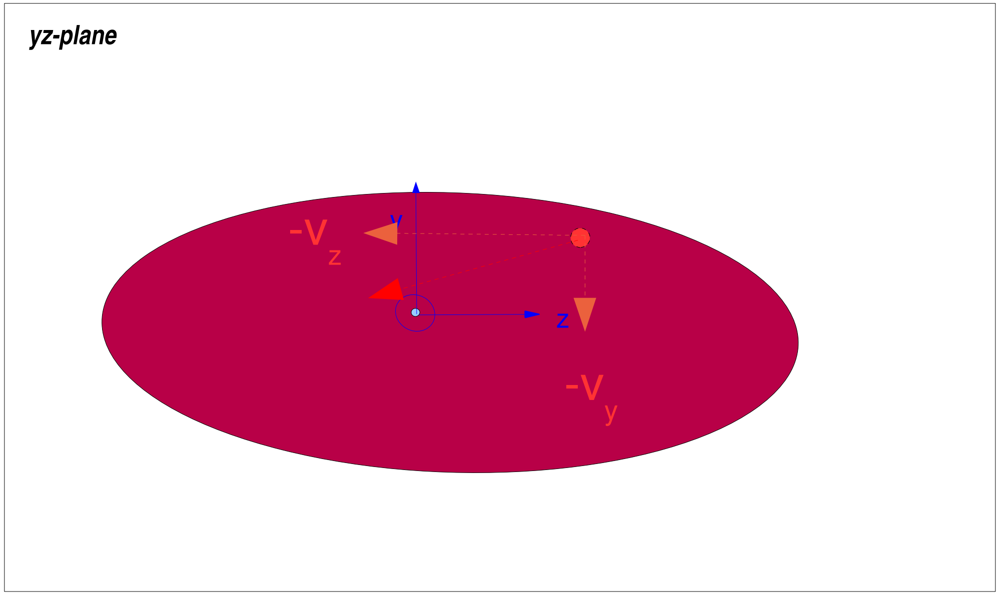


Time reversal (laboratory frame)

$$\begin{pmatrix} \vec{r}(t) \\ \vec{v}(t) \\ \vec{E}(t) \\ \vec{B}(t) \end{pmatrix} \xrightarrow{T\ :\ t\rightarrow(-t)} \begin{pmatrix} \vec{r}(t) \\ -\vec{v}(t) \\ \vec{E}(t) \\ -\vec{B}(t) \end{pmatrix}$$





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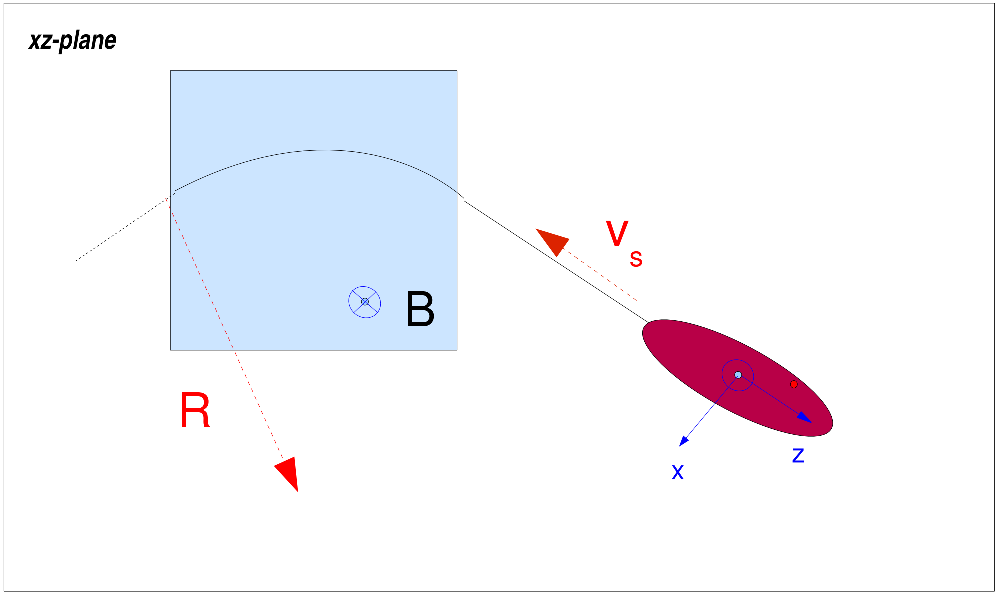

conventions:

- (x,y,z)-> left handed, moving frame
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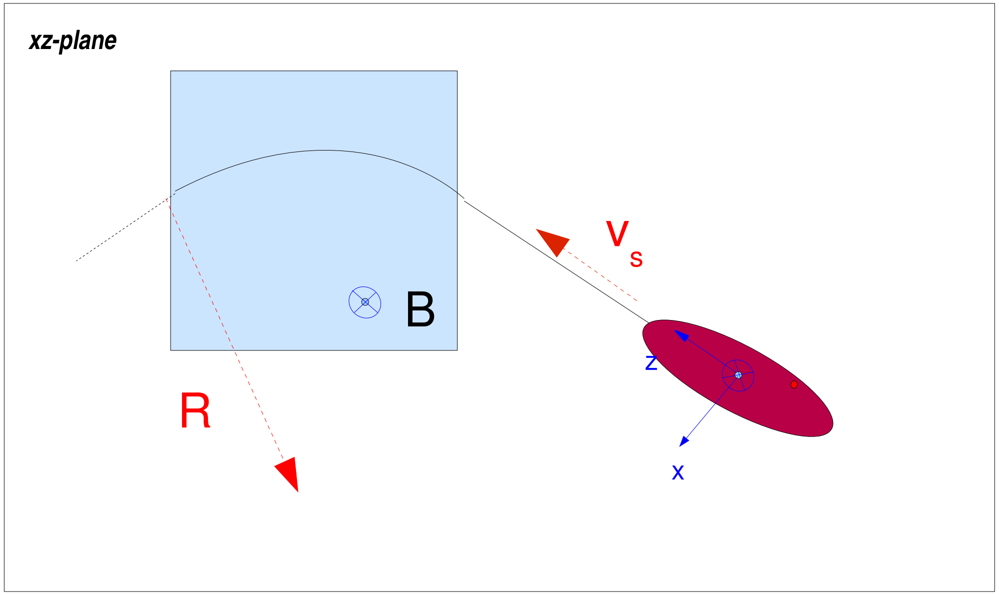
180*deg* rotation of the moving frame about the x-axis

$$M_{rot} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\pi) & \sin(\pi) \\ 0 & -\sin(\pi) & \cos(\pi) \end{pmatrix}$$

Conclusion for Beam Dynamics: „Moving Frame“



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Conclusion for Beam Dynamics: „Time Reversal“

Time reversal (laboratory frame)

$$\begin{pmatrix} \vec{r}(t) \\ \vec{v}(t) \\ \vec{E}(t) \\ \vec{B}(t) \end{pmatrix} \xrightarrow{T\ :\ t \to (-t)} \begin{pmatrix} \vec{r}(t) \\ -\vec{v}(t) \\ \vec{E}(t) \\ -\vec{B}(t) \end{pmatrix}$$

Rotation of the moving frame:

$$M_{rot} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\Rightarrow$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \xrightarrow{T} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \xrightarrow{M_{rot}} \begin{pmatrix} x \\ -y \\ -z \end{pmatrix}$$

$$\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} \xrightarrow{T} \begin{pmatrix} -v_x \\ -v_y \\ -v_z \end{pmatrix} \xrightarrow{M_{rot}} \begin{pmatrix} -v_x \\ v_y \\ v_z \end{pmatrix}$$

$$\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \xrightarrow{T} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \xrightarrow{M_{rot}} \begin{pmatrix} E_x \\ -E_y \\ -E_z \end{pmatrix}$$

$$\begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} \xrightarrow{T} \begin{pmatrix} -B_x \\ -B_y \\ -B_z \end{pmatrix} \xrightarrow{M_{rot}} \begin{pmatrix} -B_x \\ B_y \\ B_z \end{pmatrix}$$

Conclusion for Beam Dynamics: „Time Reversal“

Time reversal (laboratory frame)

$$\begin{pmatrix} \vec{r}(t) \\ \vec{v}(t) \\ \vec{E}(t) \\ \vec{B}(t) \end{pmatrix} \xrightarrow{T\ :\ t \to (-t)} \begin{pmatrix} \vec{r}(t) \\ -\vec{v}(t) \\ \vec{E}(t) \\ -\vec{B}(t) \end{pmatrix}$$

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$$\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \xrightarrow{T} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \xrightarrow{M_{rot}} \begin{pmatrix} E_x \\ -E_y \\ -E_z \end{pmatrix}$$

$$\begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} \xrightarrow{T} \begin{pmatrix} -B_x \\ -B_y \\ -B_z \end{pmatrix} \xrightarrow{M_{rot}} \begin{pmatrix} -B_x \\ B_y \\ B_z \end{pmatrix}$$

There is no change in direction of the dipole-field with respect to the moving frame

Conclusion for Beam Dynamics: „Time Reversal“

Time reversal (laboratory frame)

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Rotation of the moving frame:

$$M_{rot} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \xrightarrow{T} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \xrightarrow{M_{rot}} \begin{pmatrix} x \\ -y \\ -z \end{pmatrix}$$

$$\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} \xrightarrow{T} \begin{pmatrix} -v_x \\ -v_y \\ -v_z \end{pmatrix} \xrightarrow{M_{rot}} \begin{pmatrix} -v_x \\ v_y \\ v_z \end{pmatrix}$$

$$\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \xrightarrow{T} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \xrightarrow{M_{rot}} \begin{pmatrix} E_x \\ -E_y \\ -E_z \end{pmatrix}$$

$$\begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} \xrightarrow{T} \begin{pmatrix} -B_x \\ -B_y \\ -B_z \end{pmatrix} \xrightarrow{M_{rot}} \begin{pmatrix} -B_x \\ B_y \\ B_z \end{pmatrix}$$

How to transform space charge force?

Conclusion for Beam Dynamics: „Space Charge Force"

$$\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \xrightarrow{T} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \xrightarrow{M_{rot}} \begin{pmatrix} E_x \\ -E_y \\ -E_z \end{pmatrix}$$

?

$$\vec{F}_{1,2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3} \cdot (\vec{r}_1 - \vec{r}_2)$$

$$(\vec{r}_1 - \vec{r}_2) = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} - \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} \xrightarrow{T \times M} \begin{pmatrix} x_1 \\ -y_1 \\ -z_1 \end{pmatrix} - \begin{pmatrix} x_2 \\ -y_2 \\ -z_2 \end{pmatrix} = \begin{pmatrix} x_1 - x_2 \\ -(y_1 - y_2) \\ -(z_1 - z_2) \end{pmatrix}$$

$$\vec{F}_{12} = \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} \xrightarrow{T \times M} \begin{pmatrix} F_x \\ -F_y \\ -F_z \end{pmatrix}$$

- Consistant with **F** = q*$\mathbf{E}$
- Forces be transformed correctly by transforming the positions

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Test with LORASR: „Coordinate System“

Coordinate System :

- Left handed, moving frame
- (x, xp), (y, yp), (ϕ, dW)

$$\phi = \frac{z}{\beta\lambda}\cdot 360 \xrightarrow{\text{T x M}} \frac{-z}{\beta\lambda}\cdot 360 = -\phi$$

$$\left.\begin{aligned} xp &\approx \frac{v_x}{v_z} \\ yp &\approx \frac{v_y}{v_z} \end{aligned}\right\} \xrightarrow{\text{T x M}} \left\{\begin{aligned} \frac{-v_x}{v_z} &\approx -xp \\ \frac{v_y}{v_z} &\approx yp \end{aligned}\right.$$

Test with LORASR: „Coordinate System“

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„forward“

Input $\begin{Bmatrix} (x_0, xp_0) \\ (y_0, yp_0) \\ (\phi_0, dW_0) \end{Bmatrix}$ → **Transport** → $\begin{Bmatrix} (x, xp) \\ (y, yp) \\ (\phi, dW) \end{Bmatrix}$ Output

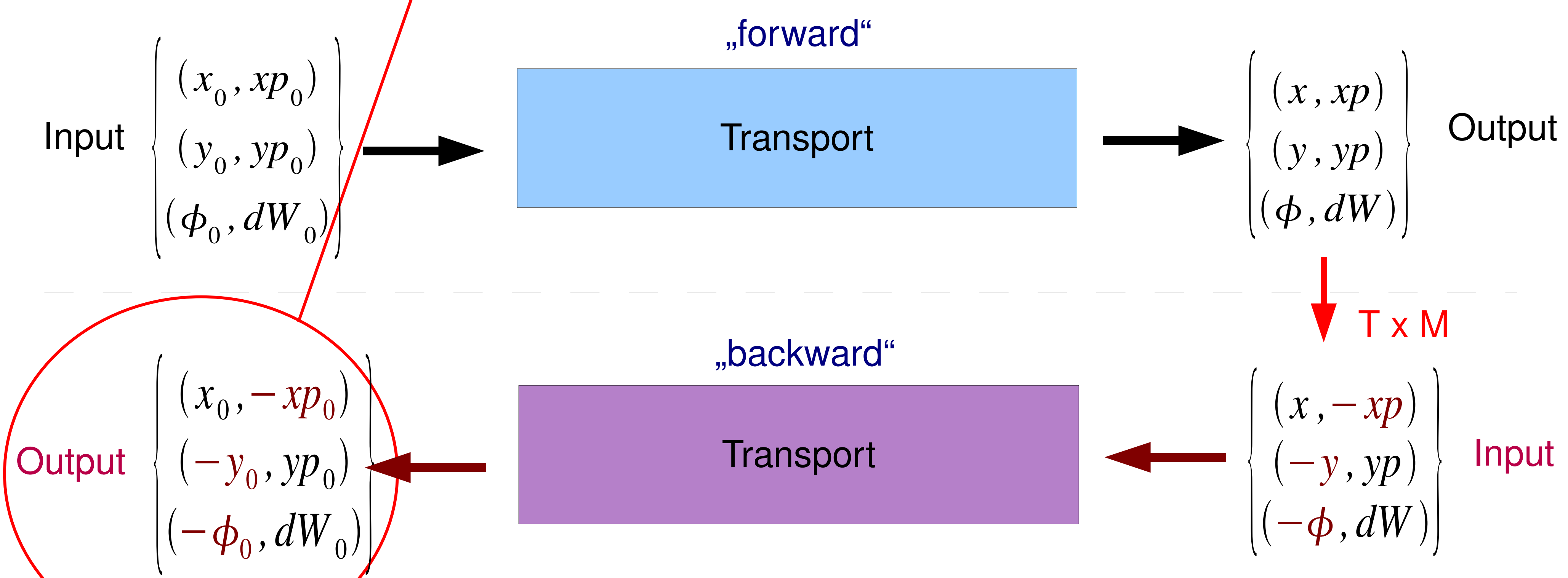
↓ T x M

„backward“

Output $\begin{Bmatrix} (x_0, -xp_0) \\ (-y_0, yp_0) \\ (-\phi_0, dW_0) \end{Bmatrix}$ ← **Transport** ← $\begin{Bmatrix} (x, -xp) \\ (-y, yp) \\ (-\phi, dW) \end{Bmatrix}$ Input

Test with LORASR: „Test Property for Runs“

$$\Delta\epsilon_{RMS,i} = \frac{\epsilon^{r,\mathrm{out}}_{RMS,i} - \epsilon^{h,\mathrm{in}}_{RMS,i}}{\epsilon^{h,\mathrm{in}}_{RMS,i}} \qquad with \quad i \in \{x, y, z\}$$



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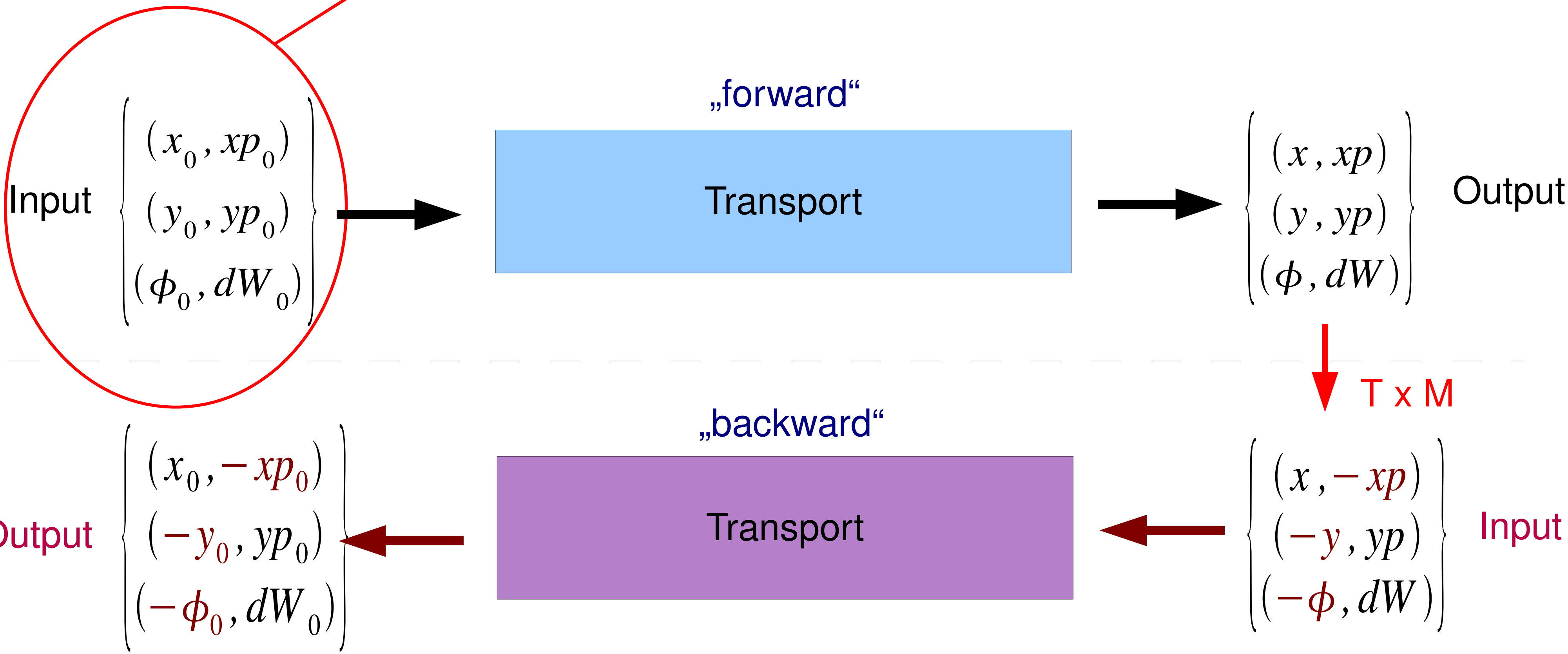


Test with LORASR: „Input Parameters“

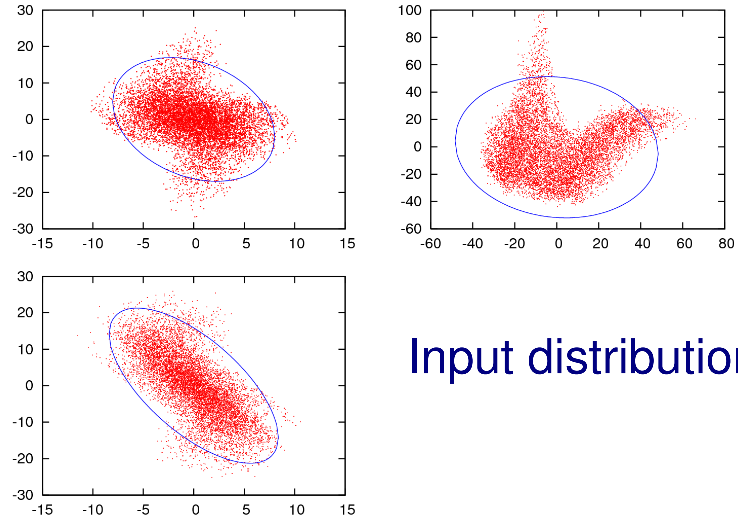
Beamline: Drift - Gap -Drift

Injectionsenergy	2.070	[MeV]
Frequency	175	[MHz]
$\beta\lambda$	0.114	[m]
Current	0.150	[mA]

Gap-Type: 1

Eff. gap voltage: 1E-20 [MV]

D/L	0.5	(Drift tube length / periode length)
G/D	0.1	(Gap length / Diameter)



Input distribution

For test runs a definite transport ***without particle losses*** is needed

$$\text{Gap-Type}(1) \Rightarrow G = \frac{\beta\lambda}{4}$$

$$D = \frac{G}{0.1} = \frac{\beta\lambda}{4\cdot 0.1} = 28.8\,cm \Rightarrow Aperture(global) = \pm 14.4\text{cm}$$

Test with LORASR: „DRIFT“

```
GRUN  GAP NO.=1,SECTIONS= 1,STRUCTURE= 1,MASS= 1,CHARGE= 1
    FREQUENCY= 175.0, PART.NO.=10000,CUP CURRENT/A= 0.150
    DRIFT BETW. SP. CH. CALLS/CM= 0.1, TRANSV. CUBE NO.= 64, NDIST=3 NFM=0
    DRIFT= L/2[cm],GAP NO.=1,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0
    DRIFT= L/2[cm}

DIST     0.0024 -7.3659 -0.7725 -0.0030 -0.3344 -0.0021
        ...
VOLT
    1E-20,1.000,1
    1.000

DDLV  D/L-RATIO=0.50,1
          0.50,1

GADI  G/D-RATIO=0.1,0.1,0.1 ...
```

L = 50 ..300 [cm]

$$Aperture(global) = \pm 14.4[cm]$$

LORASR:
At least *one gap* with marginal eff. Voltage 1E-20 *MV*

For the test runs definite transport without particle losses is needed

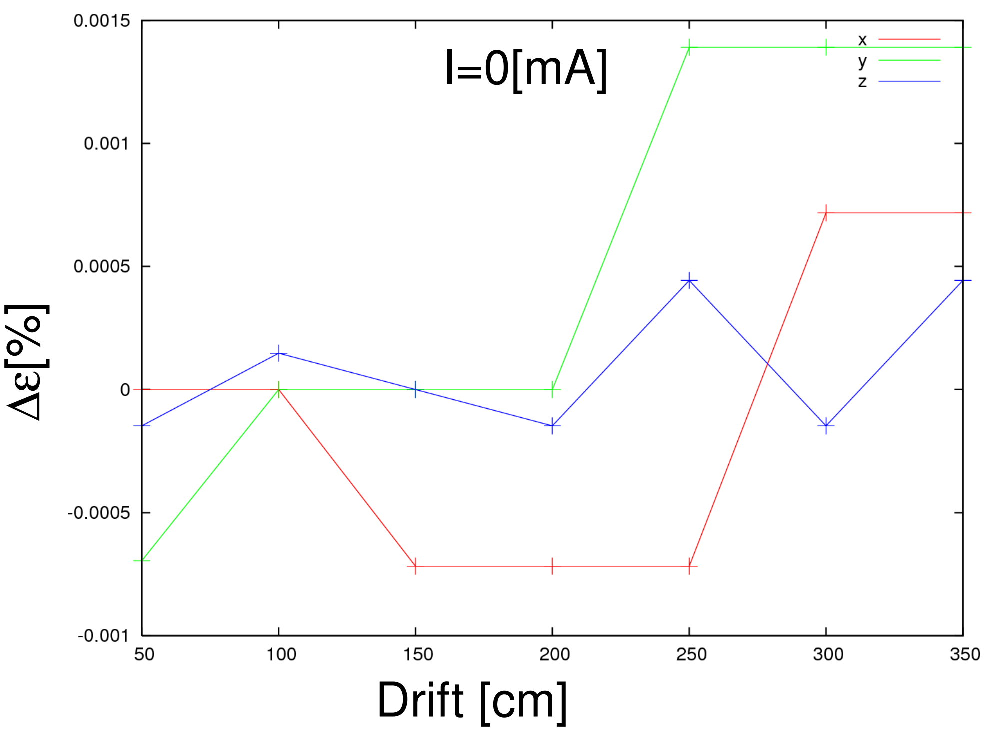
Test with LORASR: „DRIFT“

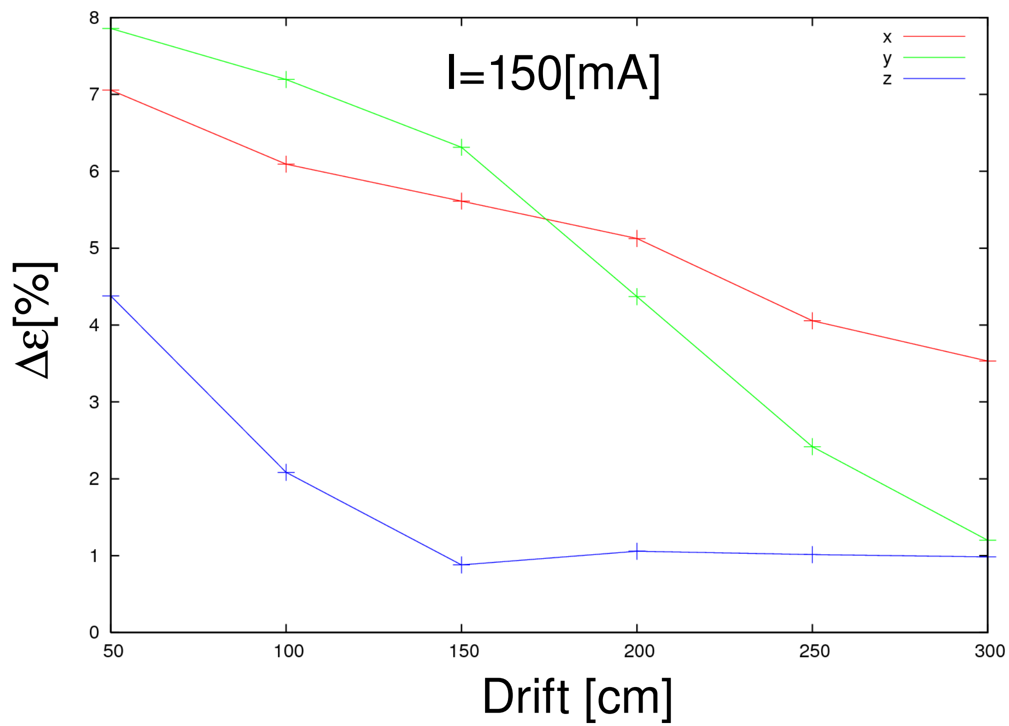
```
GRUN  GAP NO.=1,SECTIONS= 1,STRUCTURE= 1,MASS= 1,CHARGE= 1
      FREQUENCY= 175.0, PART.NO.=10000,CUP CURRENT/A= 0.150
      DRIFT BETW. SP. CH. CALLS/CM= 0.1, TRANSV. CUBE NO.= 64, NDIST=3 NFM=0
      DRIFT= L/2[cm],GAP NO.=1,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0
      DRIFT= L/2[cm]

DIST     0.0024 -7.3659 -0.7725 -0.0030 -0.3344 -0.0021
          ...
VOLT
      1E-20,1.000,1
      1.000
```

L = 50 ..300 [cm]

LORASR:
At least *one gap* with marginal eff. Voltage 1E-20 *MV*

For the test runs definite
transport without particle





Test with LORASR: „Space Charge Calculation"

LORASR-Code:
SUBROUTINE PPINT
(space charge calculation with fftw)

Grid No. :

Part. No. ≤ 10k => Nx= Ny= Nz= 32
Part. No. ≤ 100k => Nx= Ny= Nz= 64
Part. No. ≤ 1000k => Nx= Ny= Nz=128

Drift-Section (with Gap)

Lattice-Length (static)

Lx= Ly= Aperture = Gap-Diameter

Lz= { βλ /4 <=> ±45 [deg]: Part. No. ≤ 10k ; βλ /2 <=> ±90 [deg]: ***else*** }

Drift-Section (without Gap)

Lattice-Length (dynamic)

Lx= Ly= 6*Xmax = 3* Bunch-Diameter

Lz= { (6*Zmax)/2 : Part. No. ≤ 10k ; 6*Zmax : ***else*** }

Test with LORASR: „Space Charge Calculation“

LORASR-Code:
SUBROUTINE PPINT
(space charge calculation with fftw)

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Drift-Section (with Gap)

Lattice-Length (static)

Lx= Ly= Aperture = Gap-Diameter

$$L_z = \begin{cases} \beta\lambda/4 \Leftrightarrow \pm 45\,[\text{deg}]: & \text{Part. No.} \le 10\text{k} \\ \beta\lambda/2 \Leftrightarrow \pm 90\,[\text{deg}]: & \textit{else} \end{cases}$$

reasonable definition for ***LINAC design***

Drift-Section (without Gap)

Lattice-Length (dynamic)

Lx= Ly= 6*Xmax = 3* Bunch-Diameter

$$L_z = \begin{cases} (6 \cdot Z_{max})/2 & : \quad \text{Part. No.} \le 10\text{k} \\ 6 \cdot Z_{max} & : \quad \textit{else} \end{cases}$$

reasonable definition for
symmetrical bunches without halo

Test with LORASR: „Space Charge Calculation“

LORASR-Code:
SUBROUTINE PPINT
(space charge calculation with fftw)

Grid No. :

Part. No. ≤ 10k => Nx= Ny= Nz= 32
Part. No. ≤ 100k => Nx= Ny= Nz= 64
Part. No. ≤ 1000k => Nx= Ny= Nz=128

Drift-Section (with Gap)	*Drift-Section (without Gap)*
Lattice-Length (static)	***Lattice-Length (dynamic)***
Lx= Ly= Aperture = Gap-Diameter	Lx= Ly= 6*Xmax = 3* Bunch-Diameter
Lz= { βλ /4 <=> ±45 [deg]: Part. No. ≤ 10k	Lz= { (6*Zmax)/2 : Part. No. ≤ 10k
βλ /2 <=> ±90 [deg]: ***else***	6*Zmax : ***else***

Particles outside of the lattice are not taken into account for space charge calculation

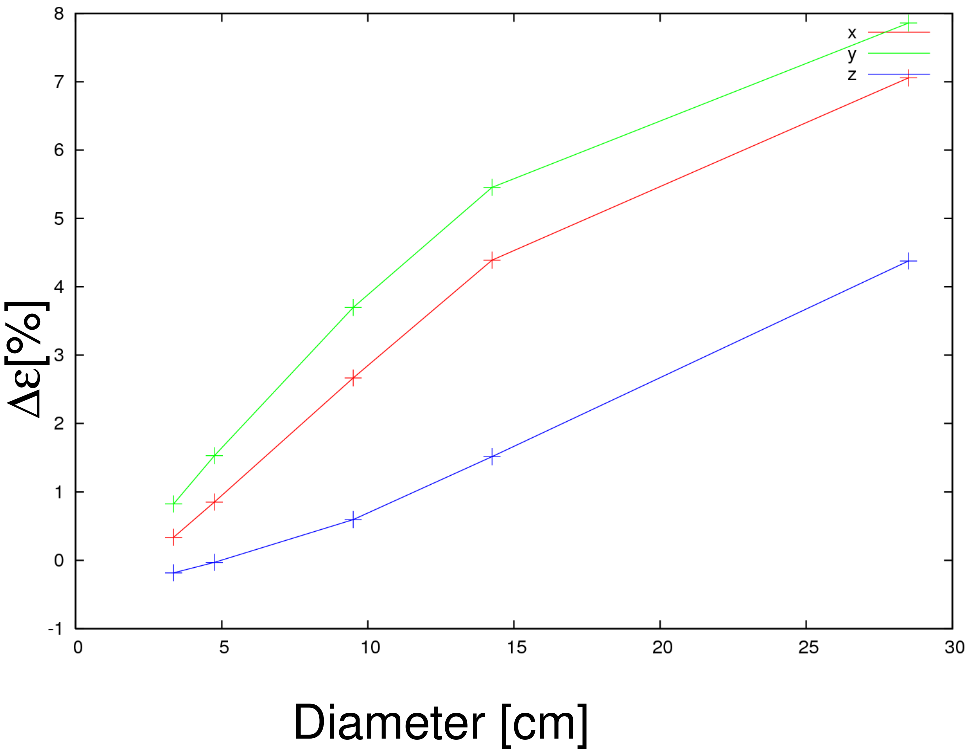
=> lesser space charge

Test with LORASR: „Aperture“

Drift: 50[cm]

G/D: 0.1 bis 0.85

(Gap length / Diameter)

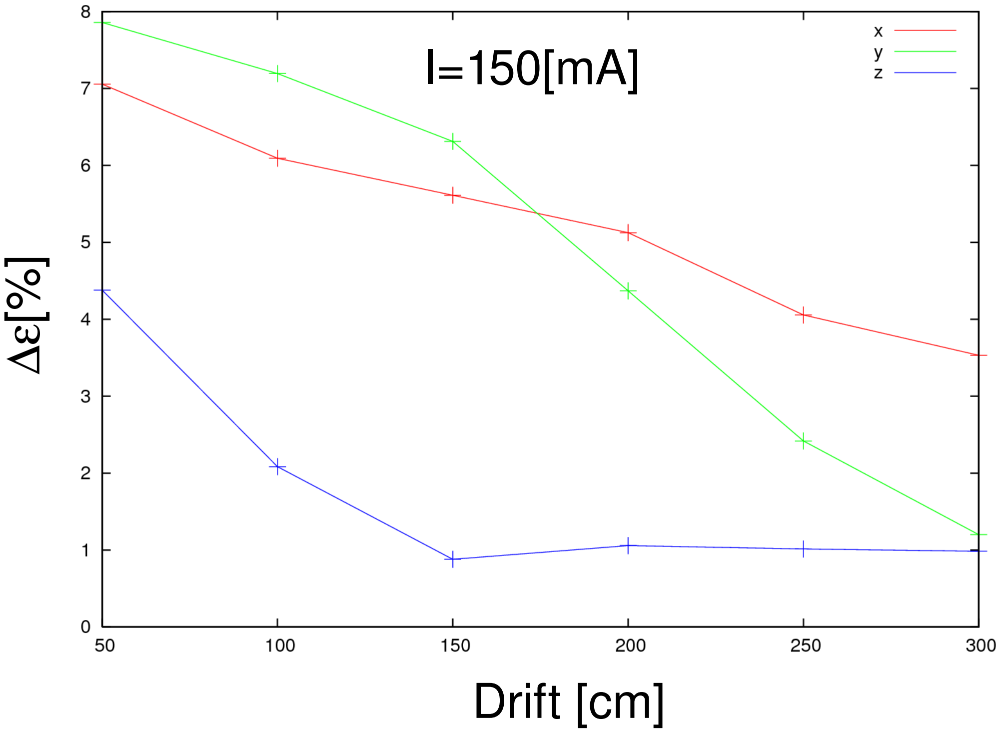


Transversal Grid:

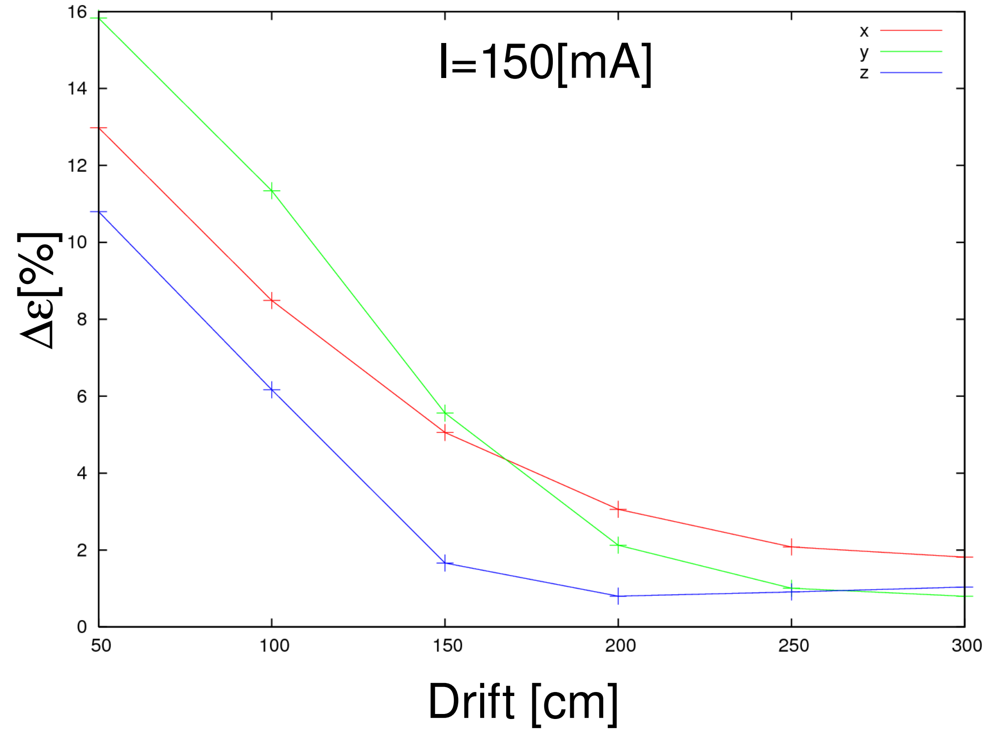
- Automatically choosen by LORASR
- Equidistant inside of the aperture
- Grid No. depent on the Particle No.:
  - N ≤ 10k => trans. Cube = 32
  - N ≤ 100k => trans. Cube = 64
  - N ≤ 1000k => trans. Cube = 128

Test with LORASR: „DRIFT“

Part. No.= 10000



Part. No.= 10000+1



Grid No.:	32
Lx/Ly/Lz:	28.80 / 28.80 / 2.85
gx/gy/gz:	0.90 / 0.90 / 0.09

Grid No.:	64
Lx/Ly/Lz:	28.80 / 28.80 / 5.70
gx/gy/gz:	0.45 / 0.45 / 0.09

Test with LORASR: „DRIFT 2“

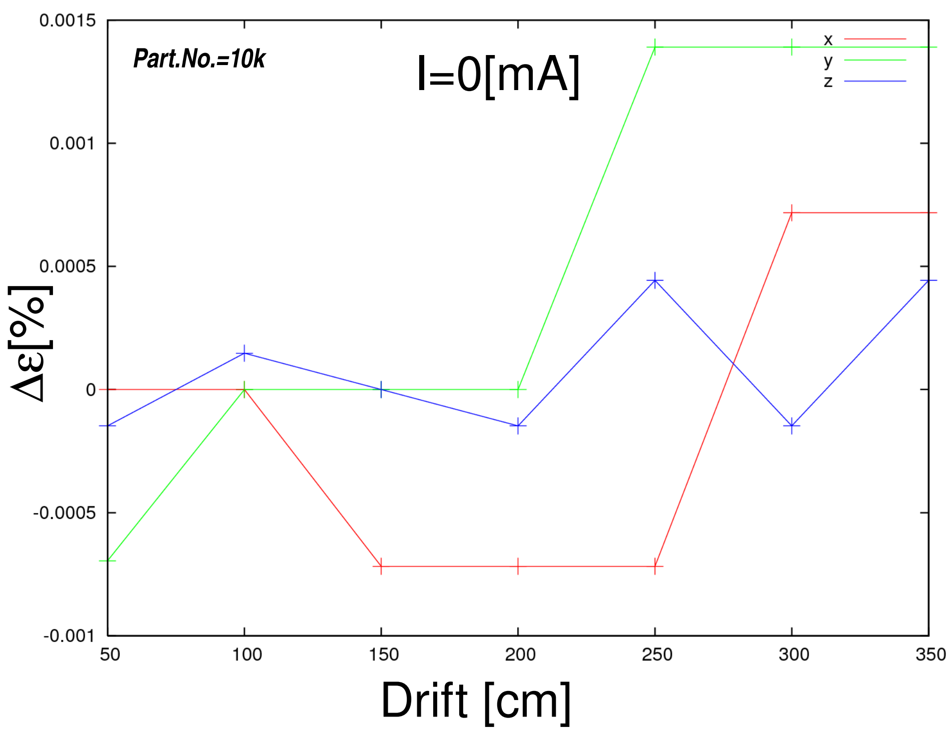
```
GRUN  GAP NO.=1,SECTIONS= 2,STRUCTURE= 1,MASS= 1,CHARGE= 1
     FREQUENCY= 175.0, PART.NO.=10000,CUP CURRENT/A= 0.150
     DRIFT BETW. SP. CH. CALLS/CM= 0.1, TRANSV. CUBE NO.= 64, NDIST=3 NFM=0
     DRIFT= L/2[cm],GAP NO.=0,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0
     DRIFT= 0.01,GAP NO.=1,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0
     DRIFT= L/2[cm]}

DIST    0.0024 -7.3659 -0.7725 -0.0030 -0.3344 -0.0021
         ...
VOLT
     1E-20,1.000,1
     1.000

DDLV  D/L-RATIO=0.50,1
             0.50,1

GADI  G/D-RATIO=0.1,0.1,0.1 ...
```

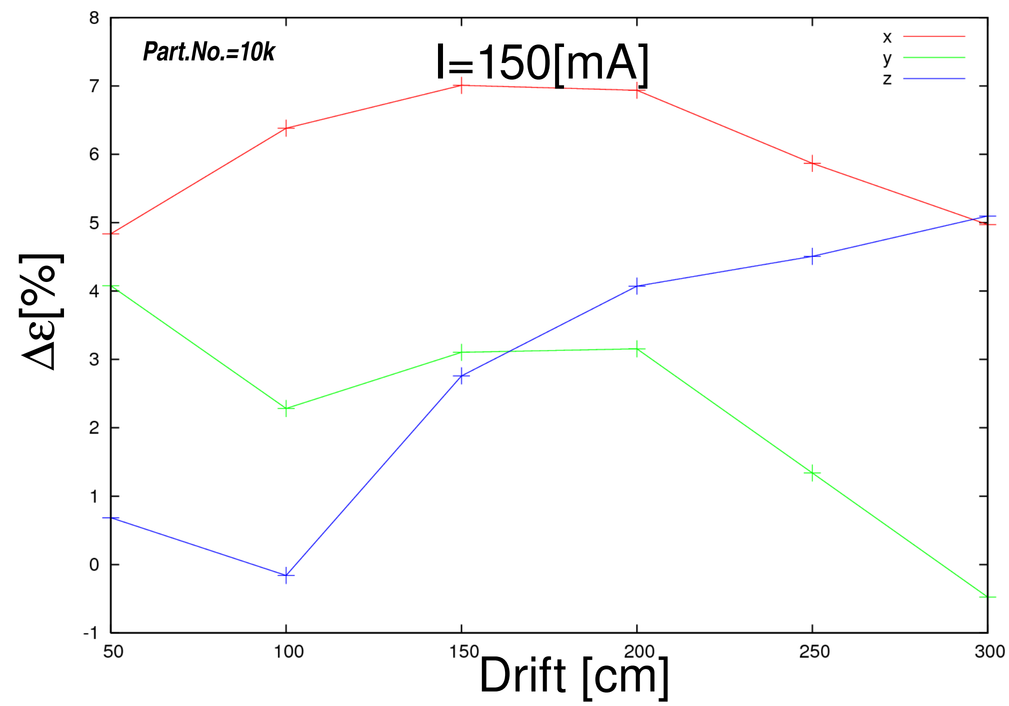
Drift section without gap.

Test with LORASR: „DRIFT 2“

```
GRUN  GAP NO.=1,SECTIONS= 2,STRUCTURE= 1,MASS= 1,CHARGE= 1
      FREQUENCY= 175.0, PART.NO.=10000,CUP CURRENT/A= 0.150
      DRIFT BETW. SP. CH. CALLS/CM= 0.1, TRANSV. CUBE NO.= 64, NDIST=3 NFM=0
      DRIFT= L/2[cm],GAP NO.=0,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0
      DRIFT= 0.01,GAP NO.=1,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0
      DRIFT= L/2[cm]

DIST     0.0024 -7.3659 -0.7725 -0.0030 -0.3344 -0.0021
```

Drift section without gap.

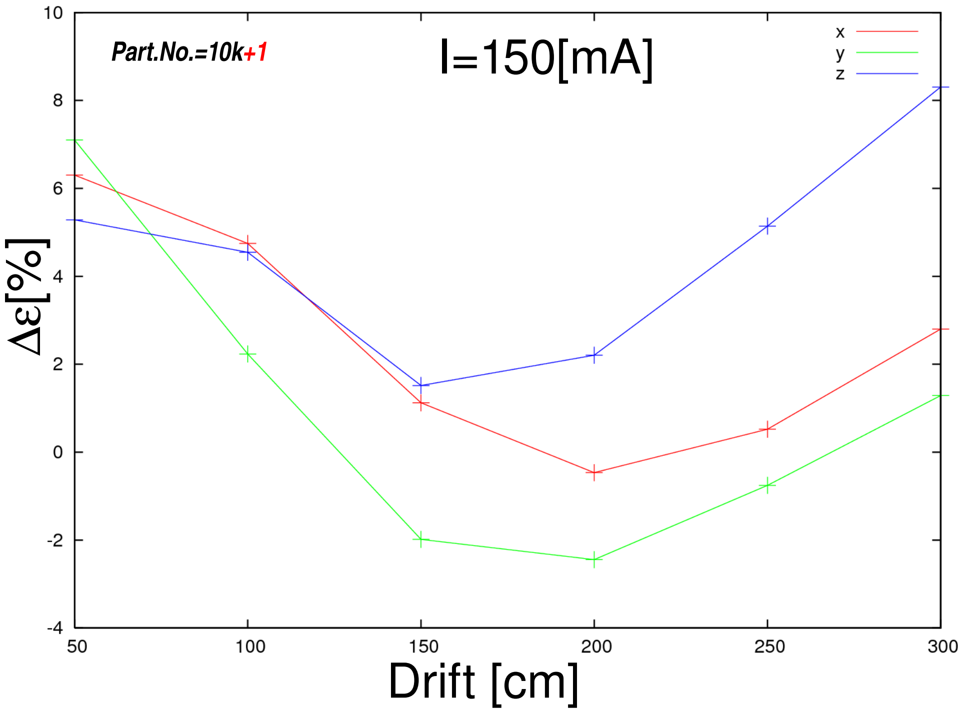


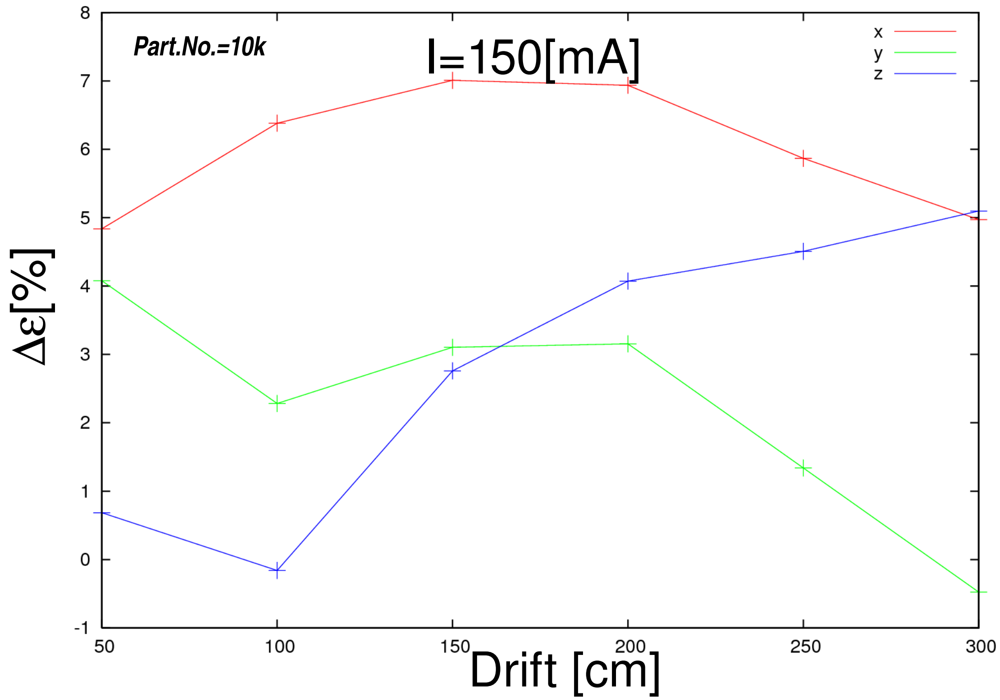


Test with LORASR: „DRIFT 2“

```
GRUN  GAP NO.=1,SECTIONS= 2,STRUCTURE= 1,MASS= 1,CHARGE= 1
      FREQUENCY= 175.0, PART.NO.=10000,CUP CURRENT/A= 0.150
      DRIFT BETW. SP. CH. CALLS/CM= 0.1, TRANSV. CUBE NO.= 64, NDIST=3 NFM=0
      DRIFT= L/2[cm],GAP NO.=0,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0
      DRIFT= 0.01,GAP NO.=1,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0
      DRIFT= L/2[cm]

DIST    0.0024 -7.3659 -0.7725 -0.0030 -0.3344 -0.0021
```

Drift section without gap.

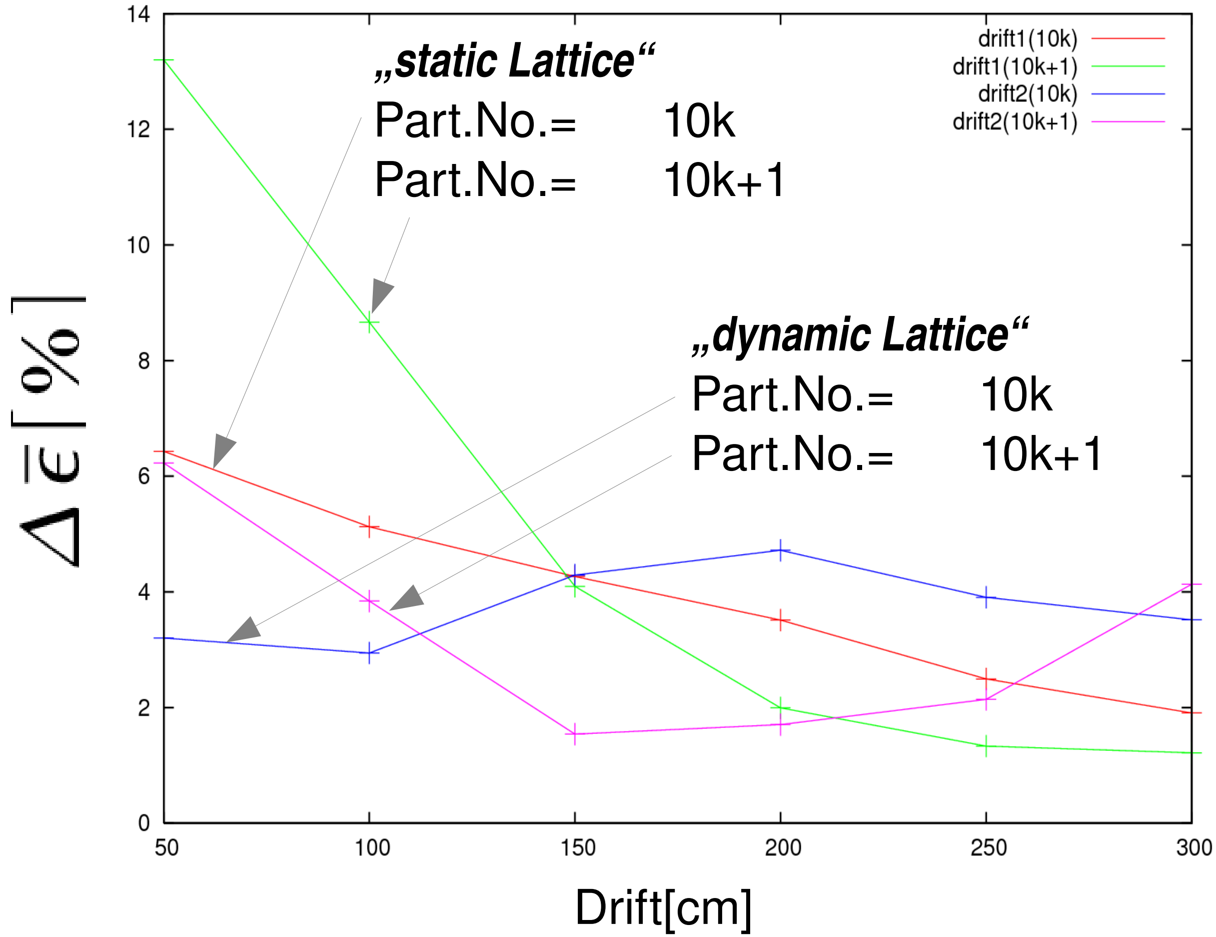




Test with LORASR: „DRIFT“ vs. „DRIFT 2“

Space charge => coppling between all planes

reasonable: $\Delta\bar{\epsilon} = \frac{1}{3} \cdot \left(|\Delta\epsilon_x| + |\Delta\epsilon_y| + |\Delta\epsilon_z| \right)$



Testlauf(„static“) ist nicht rein „static“, letzte Drift dynamisch

- Drifts < 150*cm:* dynamic lattice lead to better results.

(poss.) reason:
grid size is smaller than in static lattice

- Drifts >150*cm :* static lattice seem to be better.

(poss.) reason:
lesser particles are taken into account for space charge calulation
<=> lesser space charge

- Better results with 10k particle for drifts < 150*cm*

(poss.) reason:???

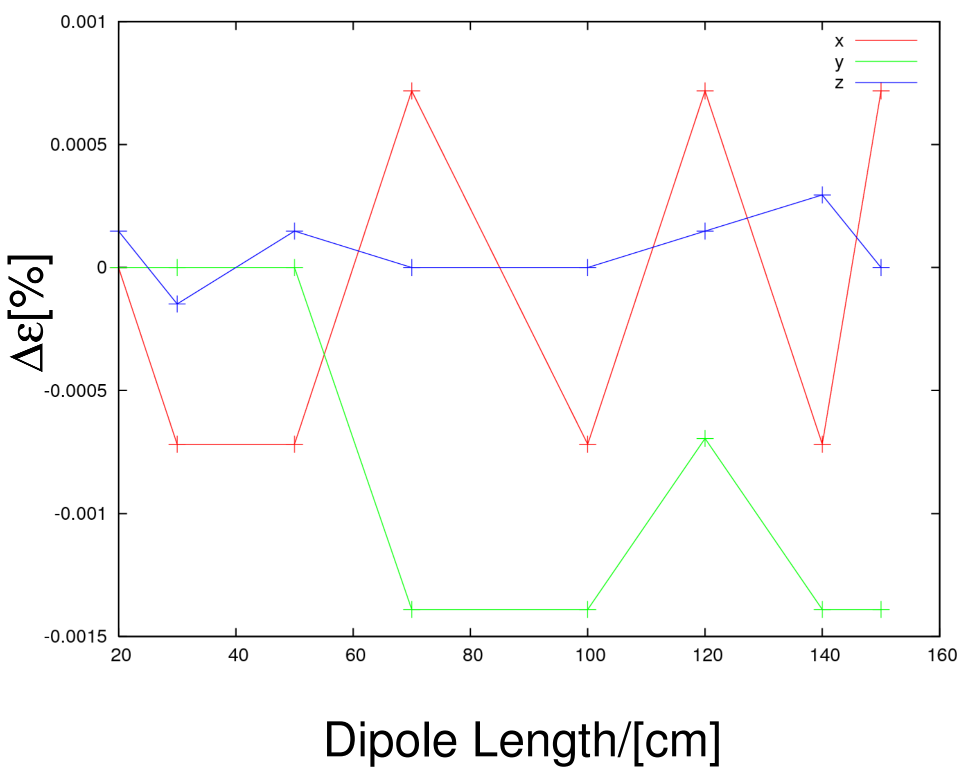
Test with LORASR: „space charge routine in DIPOLE“

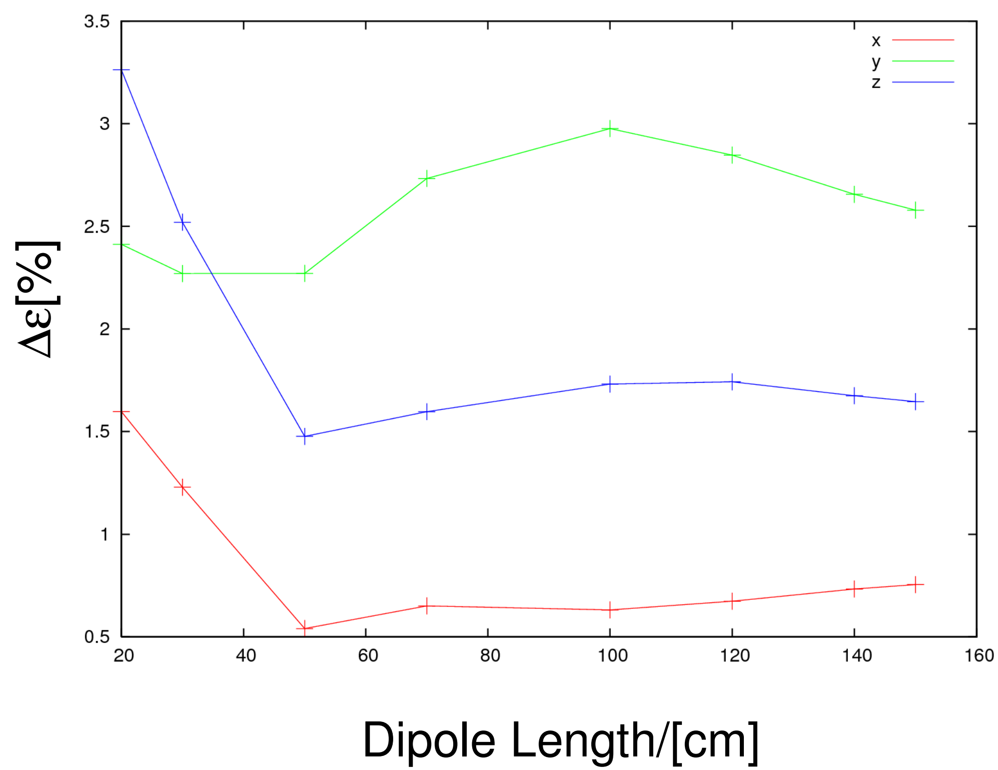
R= 2864.79[cm]   (const)

alpha= 1.0[deg]   bei   L= 50.0[cm]

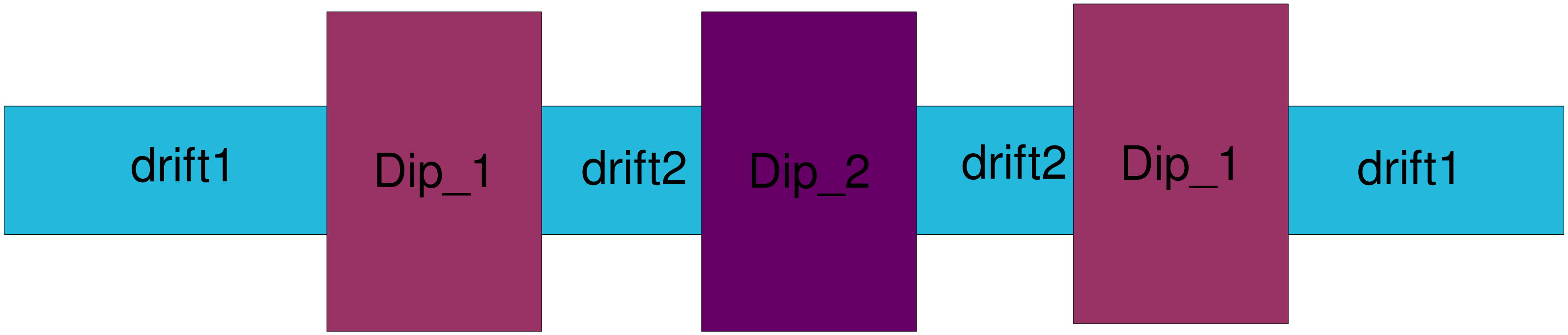
current: 0.000[A]
step size: 1[mm]

current: 0.150[A]
step size: 1[mm]





Test with LORASR: „One Trajectory of the Bunch Compressor“

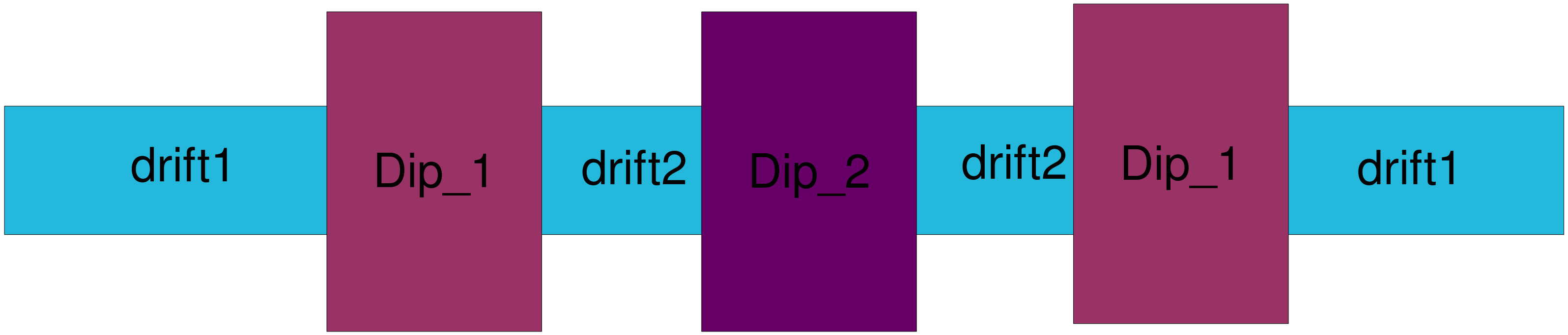



```
GRUN  GAP NO.=2 ,SECTIONS= 11,STRUCTURE= 1,MASS= 1,CHARGE= 1
    FREQUENCY= 175.0, PART.NO.=10000,CUP CURRENT/A= 0.000
    DRIFT BETW. SP. CH. CALLS/CM= 0.1, TRANSV. CUBE NO.= 64, NDIST=3 NFM=0

    DRIFT= 74.80,GAP NO.=0,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0

    DRIFT= 24.058,GAP NO.=0,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0

    DRIFT= 30.40,GAP NO.=0,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0
    DRIFT= 0.01,GAP NO.=1,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0
    DRIFT= 30.40,GAP NO.=0,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0

    DRIFT= 20.861345,GAP NO.=0,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0

    DRIFT= 30.40,GAP NO.=0,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0
    DRIFT= 0.01,GAP NO.=1,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0
    DRIFT= 30.40,GAP NO.=0,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0

    DRIFT= 24.058,GAP NO.=0,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0

    DRIFT= 74.80,GAP NO.=0,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0
    DRIFT= 0.01
```

Drift sections:

- without gaps
- dynamic lattice

Test with LORASR: „One Trajectory of the Bunch Compressor“




```
GRUN  GAP NO.=2 ,SECTIONS= 11,STRUCTURE= 1,MASS= 1,CHARGE= 1
    FREQUENCY= 175.0, PART.NO.=10000,CUP CURRENT/A= 0.000
    DRIFT BETW. SP. CH. CALLS/CM= 0.1, TRANSV. CUBE NO.= 64, NDIST=3 NFM=0

    DRIFT= 74.80,GAP NO.=0,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0

    DRIFT= 24.058,GAP NO.=0,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0

    DRIFT= 30.40,GAP NO.=0,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0
    DRIFT= 0.01,GAP NO.=1,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0
    DRIFT= 30.40,GAP NO.=0,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0

    DRIFT= 20.861345,GAP NO.=0,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0

    DRIFT= 30.40,GAP NO.=0,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0
    DRIFT= 0.01,GAP NO.=1,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0
    DRIFT= 30.40,GAP NO.=0,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0

    DRIFT= 24.058,GAP NO.=0,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0

    DRIFT= 74.80,GAP NO.=0,NFREQ.=1.0,INJ.EN.=2.070,PH.SHIFT=0.0
    DRIFT= 0.01
```

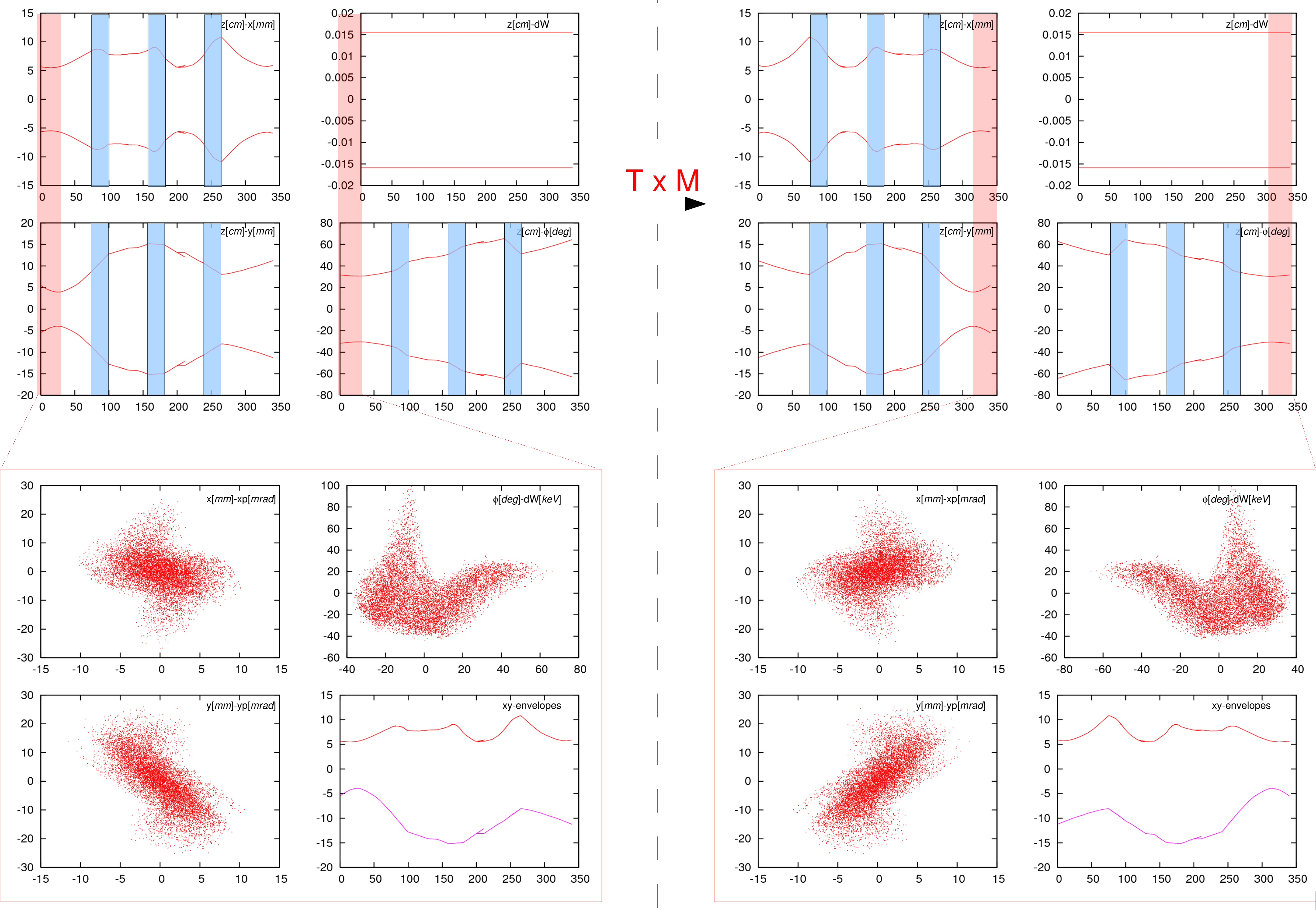
Dipole sections:

dynamic lattice

One Trajectory of the Bunch Compressor (I = 0 *mA*)

„forward“

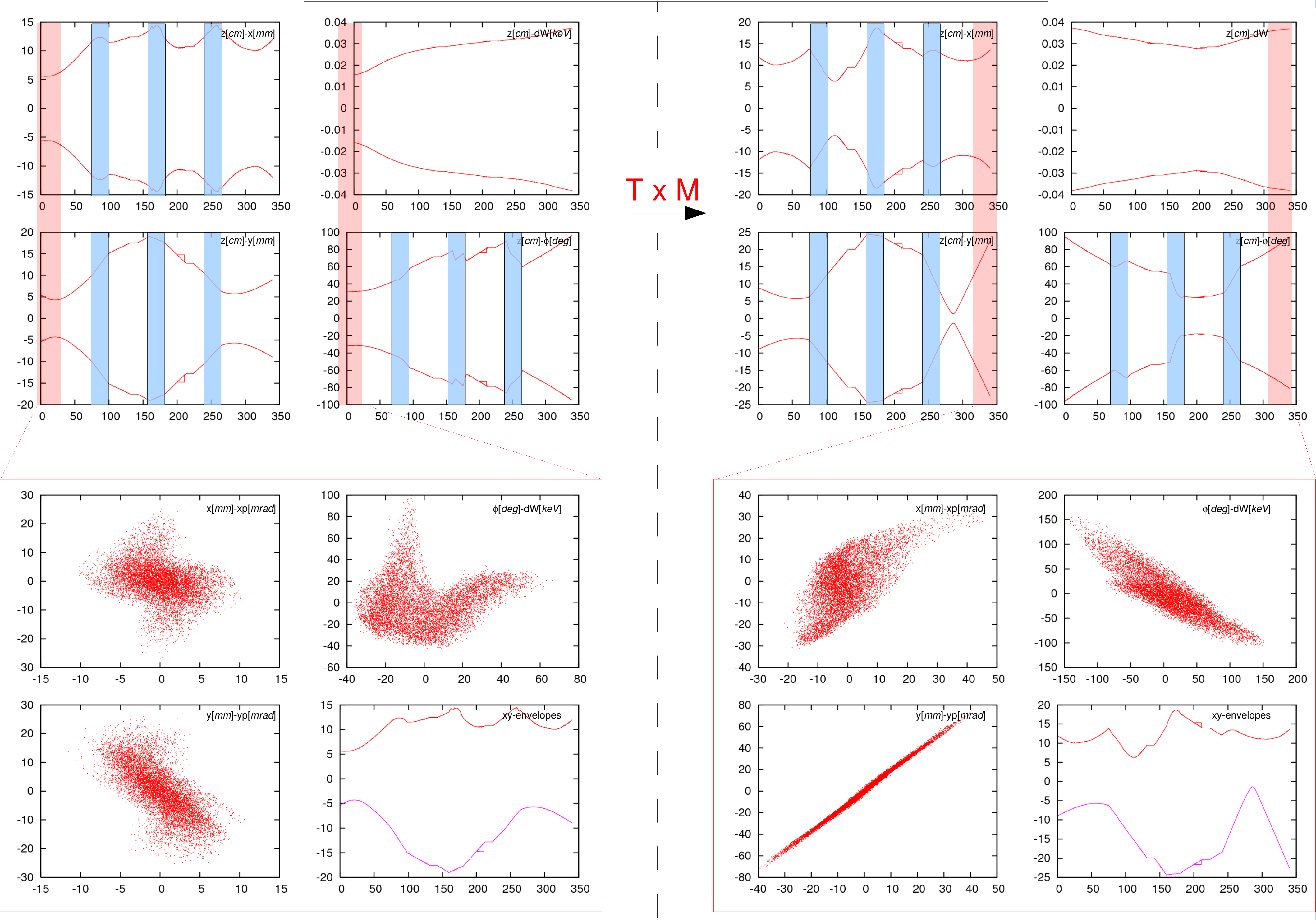
„backward“



One Trajectory of the Bunch Compressor (I = 150 ***mA***)

„forward“

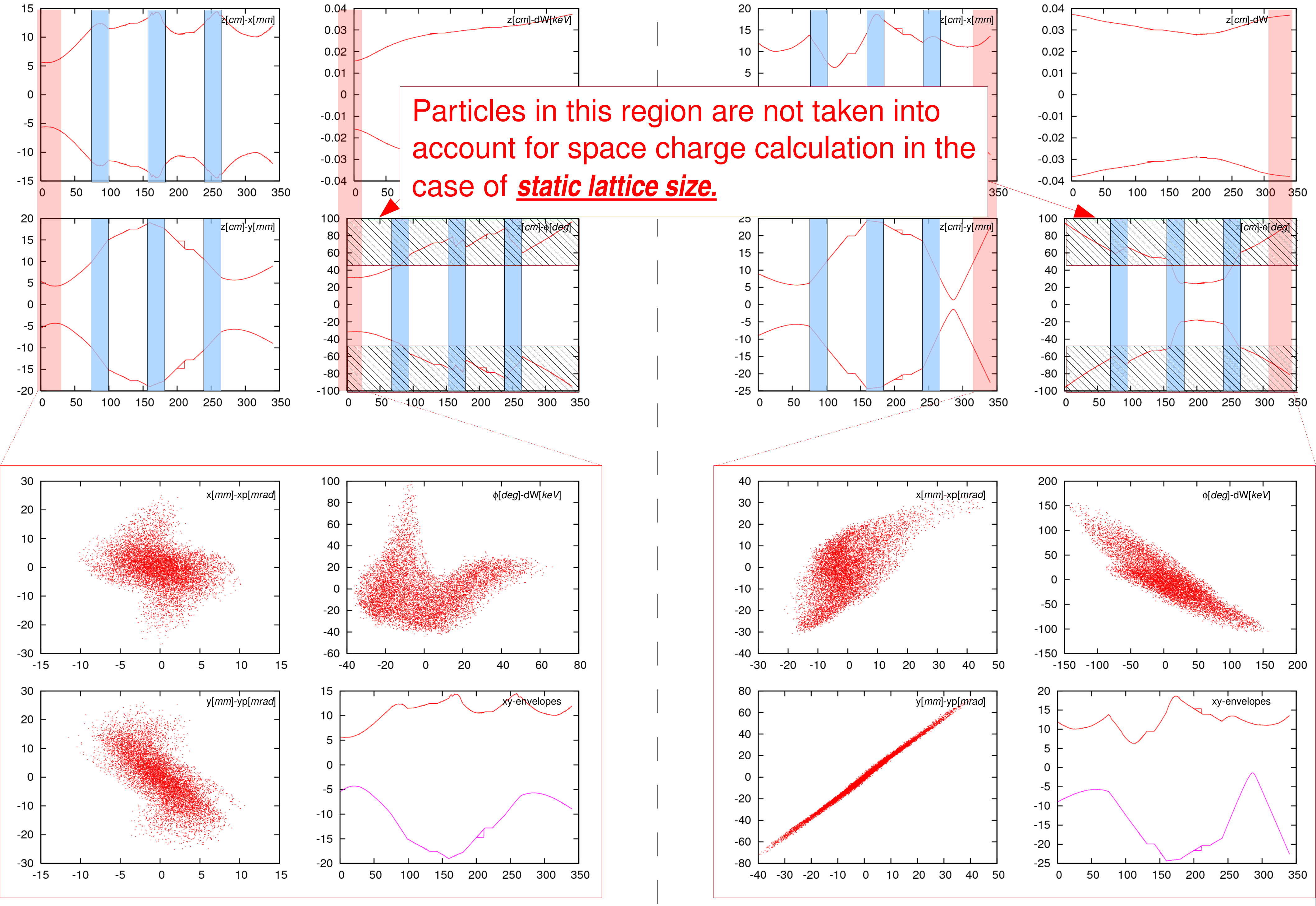
„backward“



One Trajectory of the Bunch Compressor (I = 150 *mA*)

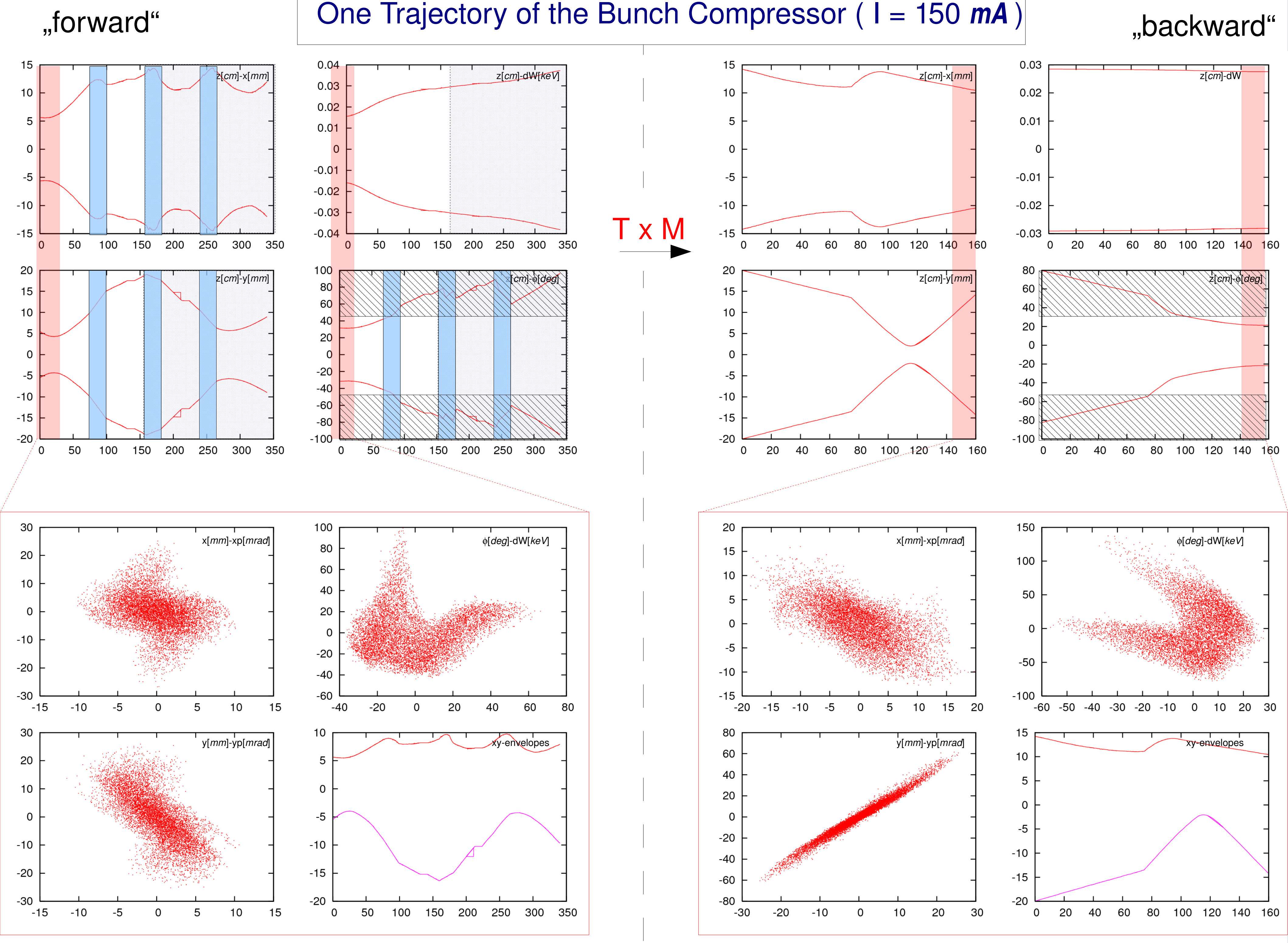
„forward“

„backward“



Particles in this region are not taken into account for space charge calculation in the case of ***static lattice size.***

One Trajectory of the Bunch Compressor (I = 150 *mA*)



Summery:

- Good results for $0mA$ beam current
- ***Geometrical parameter*** and the ***beam size*** within the bunch compressor

  => insufficient accuracy in space charge calculation (?)
- Origin of the miscalculation is not completly understood
- ***„Static lattice“*** : limitation in „Lz“ by $\beta\lambda/4$ or $\beta\lambda/2$,

  in „Lx“, „Ly“ direction by aperture
- ***„Dynamic lattice“*** : bad grid size caused by **halo-particle**

Time Reversal in Beam Dynamics / Test with LORASR

Questions:

- Are there principle ***limitations*** in „time reversal“ with ***fftw-methode***?
- ***Finite serie*** of frequencies and ***discret lattice*** for numerical calculation

  leads to ***information losses***, magnitude of information losses is not investigated yet.
- How many harmonics are needed for an accuracy < 5%

  for the bunch compressor geometry ?